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## DEPARTMENT OF BIOMEDICAL ENGINEERING

### III Semester- BM3301 SENSORS AND MEASUREMENTS

#### UNIT - 2

#### 2.1 Strain Gauge

If a metal conductor is stretched or compressed, its resistance changes on account of the fact that both length and diameter of conductor change. Also, there is a change in the value of resistivity of the conductor when it is strained and this property is called **piezoresistive effect**. Therefore, resistance strain gauges are also known as piezoresistive gauges. The strain gauges are used for measurement of strain and associated stress in experimental stress analysis. Secondly, many other detectors and transducers, notably the load cells, torque meters, diaphragm type pressure gauges, temperature sensors, accelerometers and flow meters, employ strain gauges as secondary transducers.

##### **2.1.1 Gauge Factor:**

The change in the value of resistance by straining the gauge may be partly explained by the normal dimensional behaviour of elastic material. If a strip of elastic material is subjected to tension, as shown in **Fig. 2.1** or in other words positively strained, its longitudinal dimension will increase while there will

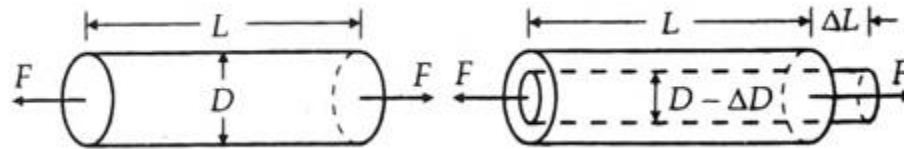


Fig. 2.1 Change in dimensions of a strain gauge element when subjected to a tensile force.

be a reduction in the lateral dimension. So when a gauge is subjected to a positive strain, its length increases while its area of cross-section decreases as shown in **Fig. 2.1**. Since the resistance of a conductor is proportional to its length and inversely proportional to its area of cross-section, the resistance of the gauge increases with positive strain. The change in the value resistance of strained conductor is more than what can be accounted for an increase in resistance due to dimensional changes. The extra change in the value of resistance is attributed to a change in the value of resistivity of a conductor when strained. This property, as described earlier, is known as **piezoresistive effect**.

Let us consider a strain gauge made of circular wire. The wire has the dimensions : length =  $L$ , area =  $A$ , diameter =  $D$  before being strained. The material of the wire has a resistivity  $\rho$ .

Resistance of unstrained gauge  $R = \rho L / A$ .

Let a tensile stress  $s$  be applied to the wire. This produces a positive strain causing the length to increase and area to decrease as shown in Fig. 2.1.

Thus, when the wire is strained there are changes in its dimensions. Let  $\Delta L$  = change in length,  $\Delta A$  = change in area,  $\Delta D$  = change in diameter and  $\Delta R$  = change in resistance.

In order to find how  $R$  depends upon the material physical quantities, the expression for  $R$  is differentiated with respect to stress  $s$ .

Thus we get:

$$\frac{dR}{ds} = \frac{\rho}{A} \frac{\partial L}{\partial s} - \frac{\rho L}{A^2} \frac{\partial A}{\partial s} + \frac{L}{A} \frac{\partial \rho}{\partial s}$$

-----(2.1)

Dividing Eqn. 2.1 throughout by resistance  $R = \rho L / A$ , we have

$$\frac{1}{R} \frac{dR}{ds} = \frac{1}{L} \frac{\partial L}{\partial s} - \frac{1}{A} \frac{\partial A}{\partial s} + \frac{1}{\rho} \frac{\partial \rho}{\partial s} \quad \text{-----(2.2)}$$

It is evident from Eqn. 2.2, that the per unit change in resistance is due to :

- (i) per unit change in length =  $\Delta L / L$ ,
- (ii) per unit change in area =  $\Delta A / A$ , and
- (iii) per unit change in resistivity =  $\Delta \rho / \rho$

Area  $A = \frac{\pi}{4} D^2 \quad \therefore \frac{\partial A}{\partial s} = 2 \cdot \frac{\pi}{4} D \cdot \frac{\partial D}{\partial s}$  -----(2.3)

$$\frac{1}{A} \frac{dA}{ds} = \frac{(2\pi/4)D}{(\pi/4)D^2} \frac{\partial D}{\partial s} = \frac{2}{D} \frac{\partial D}{\partial s} \quad \text{-----(2.4)}$$

$\therefore$  Equation 2.2 can be written as :

$$\frac{1}{R} \frac{dR}{ds} = \frac{1}{L} \frac{\partial L}{\partial s} - \frac{2}{D} \frac{\partial D}{\partial s} + \frac{1}{\rho} \frac{\partial \rho}{\partial s} \quad \text{-----(2.5)}$$

Now, Poisson's ratio

$$\nu = \frac{\text{lateral strain}}{\text{longitudinal strain}} = -\frac{\partial D / D}{\partial L / L} \quad \text{-----(2.6)}$$

or  $\partial D / D = -\nu \times \partial L / L$

$$\therefore \frac{1}{R} \frac{dR}{ds} = \frac{1}{L} \frac{\partial L}{\partial s} + \nu \frac{2}{L} \frac{\partial L}{\partial s} + \frac{1}{\rho} \frac{\partial \rho}{\partial s} \quad \text{-----(2.7)}$$

For small variations, the above relationship can be written as :

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + 2\nu \frac{\Delta L}{L} + \frac{\Delta \rho}{\rho} \quad \text{-----(2.8)}$$

**The gauge factor** is defined as the ratio of per unit change in resistance to per unit change in length.

Gauge factor  $G_f = \frac{\Delta R / R}{\Delta L / L}$  -----(2.9)

or  $\frac{\Delta R}{R} = G_f \frac{\Delta L}{L} = G_f \times \epsilon$  -----(2.10)

where  $\epsilon = \text{strain} = \frac{\Delta L}{L}$

The gauge factor can be written as :

$$= 1 + 2\nu + \frac{\Delta \rho / \rho}{\epsilon} \quad \text{-----(2.11)}$$

$= 1$	$+ 2\nu$	$+ \frac{\Delta \rho / \rho}{\epsilon}$
Resistance change due to change of length	Resistance change due to change in area	Resistance change due to piezoresistive effect

$$G_f = \frac{\Delta R / R}{\Delta L / L} = 1 + 2\nu + \frac{\Delta \rho / \rho}{\Delta L / L}$$

The strain is usually expressed in terms of microstrain. 1 microstrain = 1  $\mu\text{m} / \text{m}$ .  
If the change in the value of resistivity of a material when strained is neglected, the gauge factor is :

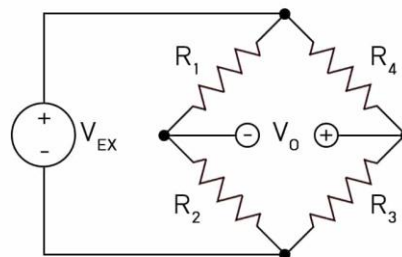
$$G_f = 1 + 2\nu$$

------(2.12)

Equation 2.12 is valid only when Piezoresistive Effect *i.e.*, change in resistivity due to strain is almost negligible.

### 2.1.2 Strain measurement using Wheatstone bridge:

A typical Wheatstone bridge circuit consists of a simple network of four resistors of equal resistances connected end to end to form a square as shown in the below figure. Across one pair of diagonal corners of the circuit, an excitation voltage is applied and across the other pair, the output of the bridge is measured. The output of the bridge, *i.e.* the value of  $V_0$  depends on the ratio of resistances of the resistors, *i.e.*  $R_1:R_4$  and  $R_2:R_3$ .



When the bridge is balanced and no strain is induced upon the strain gauge, the relationship between the four resistances can be expressed as:

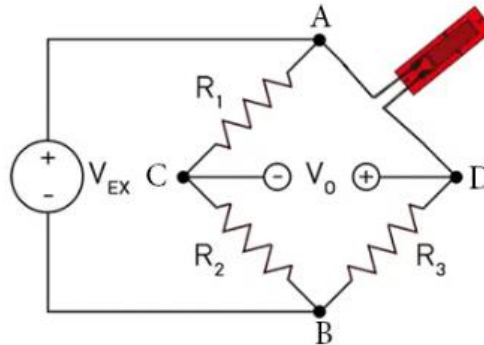
$$\frac{R_1}{R_4} = \frac{R_2}{R_3}$$

### Forms of Wheatstone Bridge Circuits

In some applications, depending on the measurement task, usually, only some of the bridge arms contain active strain gauges, the remainder consisting of bridge completion resistors. This includes bridge arrangements such as quarter bridge, half bridge, or diagonal bridge. For applications with very stringent accuracy requirements, a full bridge arrangement is preferred.

#### Quarter Wheatstone Bridge

In case of an increase in the resistance of one of the resistors in the bridge due to the applied force, the bridge no longer stays balanced. This configuration is known as a quarter bridge strain gauge.



Here,

$$\frac{R_1}{(R_1 + R_2)} \neq \frac{R_4 + \Delta R}{R_3} \text{ and } V_0 \neq 0$$

Now, using Voltage divider formula, we get

$$V_C = \frac{R_2}{(R_1 + R_2)} V_{EX}$$

$$V_D = \frac{(R_4 + \Delta R)}{(R_4 + \Delta R) + R_3} V_{EX}$$

Also, voltage drop  $V_0$  can be expressed as:

$$V_0 = \frac{(R + \Delta R)}{(R + (R + \Delta R))} V_{EX} - \frac{R}{(R + R)} V_{EX} \quad [\text{Since } R_1 = R_2 = R_3 = R_4 = R]$$

On further simplification, we get

$$V_0 = \frac{(V_{EX} \Delta R)}{(4R + 2\Delta R)}$$

Now, since  $R \gg \Delta R$  and  $4R \gg 2\Delta R$ , we get the following final equation:

$$V_0 = \frac{V_{EX}}{4} \times \frac{\Delta R}{R}$$

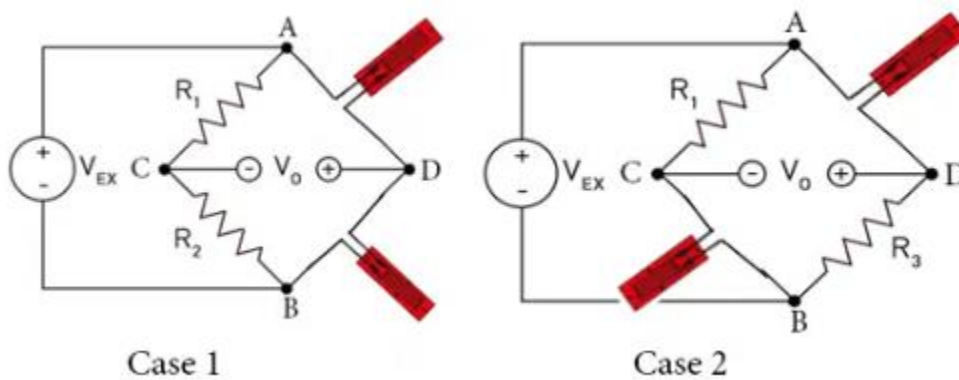
In the above equation,  $\Delta R/R$  is the electrical strain and the gauge factor is termed as the ratio of the electrical strain and the mechanical strain. Therefore, the above equation can be re-written as:

$$V_0 = \frac{V_{ex}}{4} \times k \times \epsilon$$

Where  $k$  = gauge factor  
and  $\epsilon$  = Mechanical strain

### Half Wheatstone Bridge

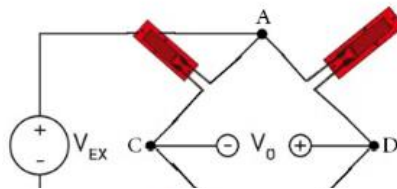
When we mount two active strain gauges on a bending beam, placed one at the front and one at the back, we get a half-bridge arrangement. This is because half of the four resistors in the circuit are now strain gauges. This arrangement will have both the strain gauges respond to the induced strain, thereby making the bridge more responsive to the applied force. As compared to the quarter bridge configuration, the half-bridge circuit yields twice the output voltage for a given strain, thereby improving the sensitivity of the circuit by a factor of two



$$V_0 = \frac{V_{ex}}{2} \times k \times \epsilon$$

### Full Wheatstone Bridge

On substituting all the resistors of the Wheatstone bridge circuit with four active strain gauges, we get a full-bridge arrangement. This configuration enables large outputs of strain-gage transducers, improves temperature compensation and offers even greater sensitivity. A full-bridge strain gauge Wheatstone bridge gives linear output than other configurations as the output voltage is directly proportional to an applied force, with no other approximation involved, unlike the quarter and half-bridge configurations.



$$V_0 = \frac{V_{EX}}{4} \times \frac{k}{4} (\epsilon_1 - \epsilon_2 + \epsilon_3 - \epsilon_4)$$

### 2.1.3 Types of Strain Gauges:

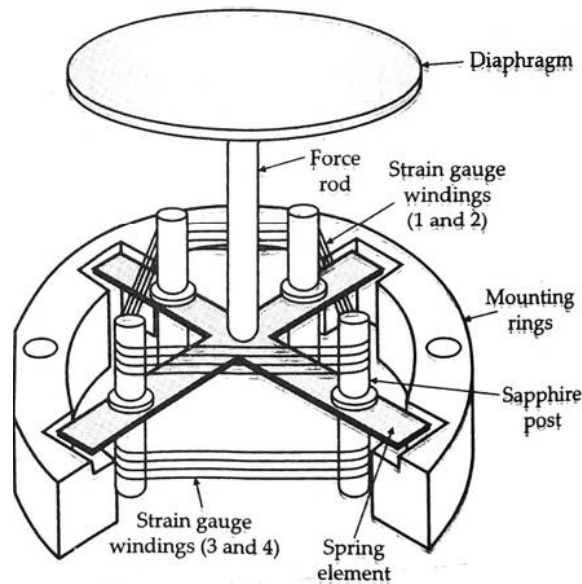
The following are the major types of strain gauges:

- Unbonded metal strain gauges
- Bonded metal wire strain gauges
- Bonded metal foil strain gauges
- Vacuum deposited thin metal film strain gauges
- Sputter deposited thin metal strain gauges
- Bonded semiconductor strain gauges
- Diffused metal strain gauges.

### 2.1.4 unbonded strain gage:

An unbonded metal strain gauge is shown in Fig. 2.2. This gauge consists of a wire stretched between two points in an insulating medium such as air. The wires may be made of various copper nickel, chrome nickel or nickel iron alloys. They are about 0.003 mm in diameter, have a gauge factor of 2 to 4 and sustain a force of 2 mN. The length of wire is 25 mm or less.





## 2.2 Set-up of a unbonded strain gauge

In Fig. 2.2, the flexure element is connected via a rod to a diaphragm which is used for sensing of pressure. The wires are tensioned to avoid buckling when they experience a compressive force. The unbonded metal wire gauges, used almost exclusively in transducer applications, employ preloaded resistance wires connected in a wheatstone bridge. At initial preload, the strains and resistances of the four arms are nominally equal, with the result the output voltage of the bridge,  $e_Q = 0$ . Application of pressure produces a small displacement which is about 0.004 mm (full scale), the displacement increases tension in two wires and decreases it in the other two, thereby increase the resistance of two wires which are in tension and decreasing the resistance of the remaining two wires. This causes an unbalance of the bridge producing an output voltage which is proportional to the input displacement and hence to the applied pressure.

### 2.1.5 Bio-medical Applications of strain gauge:

**Gait Analysis:** Strain gauges can be attached to different parts of the body to measure the strain on muscles, tendons, and bones during walking or running.

**Prosthetics and Orthotics:** Strain gauges can be incorporated into prosthetic limbs and orthotic devices to monitor the forces and strains experienced by the device during movement

**Pressure Sensors in Implants:** Strain gauges can be used as pressure sensors in medical implants such as stents or orthopedic implants.

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