SCALARS AND VECTORS

Some physical quantities such as length, area, volume and mass can be completely described by a single real number. Because these quantities are describable by giving only a magnitude, they are called **scalars**. [The word scalar means representable by position on a line; having only magnitude.] On the other hand physical quantities such as displacement, velocity, force and acceleration require both a magnitude and a direction to completely describe them. Such quantities are called **vectors**.

If you say that a car is traveling at 90 km/hr, you are using a scalar quantity, namely the number 90 with no direction attached, to describe the speed of the car. On the other hand, if you say that the car is traveling due north at 90 km/hr, your description of the car's velocity is a vector quantity since it includes both magnitude and direction.

To distinguish between scalars and vectors we will denote scalars by lower case italic type such as *a*, *b*, *c* etc. and denote vectors by lower case boldface type such as **u**, **v**, **w** etc. In handwritten script, this way of distinguishing between vectors and scalars must be modified. It is customary to leave scalars as regular hand written script and modify the symbols used to represent vectors by either underlining, such as $\underline{u}$ or $\underline{v}$, or by placing an arrow above the symbol, such as u or v

Problems

1. Determine whether a scalar quantity, a vector quantity or neither would be appropriate to describe each of the following situations.

   a. The outside temperature is 15º C.

   b. A truck is traveling at 60 km/hr.

   c. The water is flowing due north at 5 km/hr.

   d. The wind is blowing from the south.

   e. A vertically upwards force of 10 Newtons is applied to a rock.

   f. The rock has a mass of 5 kilograms.

   g. The box has a volume of .25 m^3.

   h. A car is speeding eastward.

   i. The rock has a density of 5 gm/cm^3.

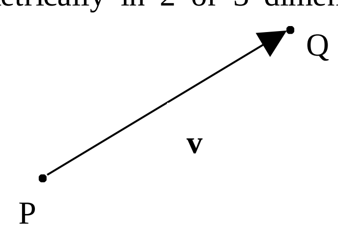
   j. A bulldozer moves the rock eastward 15m.

   k. The wind is blowing at 20 km/hr from the south.

   l. A stone dropped into a pond is sinking at the rate of 30 cm/sec.

1.2 GEOMETRICAL REPRESENTATION OF VECTORS

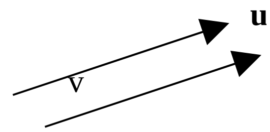
Because vectors are determined by both a magnitude and a direction, they are represented geometrically in 2 or 3 dimensional space as **directed line segments** or **arrows**. The length of the arrow corresponds to the magnitude of the vector while the direction of the arrow corresponds to the direction of the



vector. The tail of the arrow is called the **initial point** of the vector while the tip of the arrow is called the **terminal point** of the vector. If the vector **v** has the point P as its initial point and the point Q as its terminal point we will write $\mathbf{v} = \overrightarrow{PQ}$.

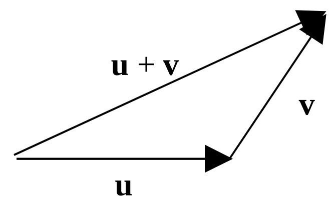
Equal vectors

Two vectors **u** and **v**, which have the same length and same direction, are said to be **equal vectors** even though they have different initial points and different terminal points. If **u** and **v** are equal vectors we write **u** = **v**.



Sum of two vectors

The **sum** of two vectors **u** and **v**, written **u** + **v** is the vector determined as follows. Place the vector **v** so that its initial point coincides with the terminal point of the vector **u.** The vector **u** + **v** is the vector whose initial point is the initial point of **u** and whose terminal point is the terminal point of **v**.



Zero vector

The **zero vector**, denoted **0**, is the vector whose length is 0. Since a vector of length 0 does not have any direction associated with it we shall agree that its direction is arbitrary; that is to say it can be assigned any direction we choose. The zero vector satisfies the property: **v** + **0** = **0** + **v** = **v** for every vector **v**.

Negative of a vector

If **u** is a nonzero vector, we define the **negative of u**, denoted **–u**, to be the vector whose magnitude (or length) is the same as the magnitude (or length) of the vector **u,** but whose direction is opposite to that of **u**.

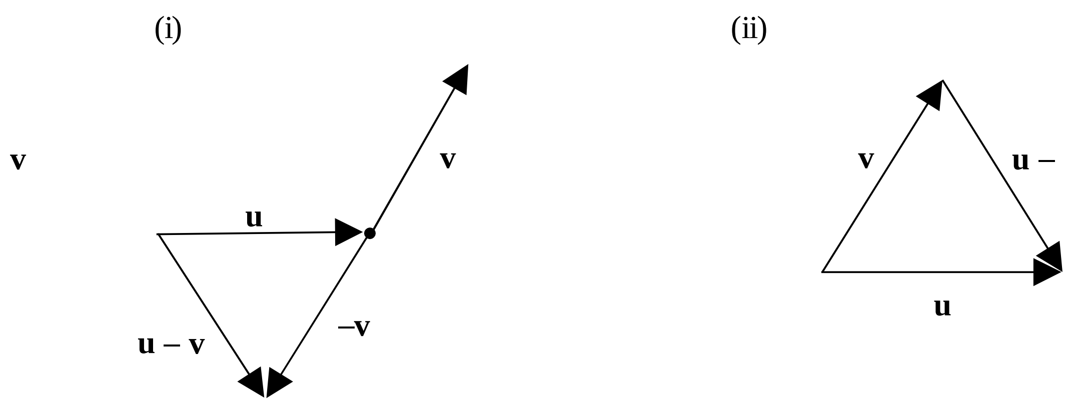
If AB is used to denote the vector from point A to point B, then the vector from point B to point A is denoted by BA, and $BA = -AB$.

Difference of two vectors

If **u** and **v** are any two vectors, we define the **difference of u and v**, denoted **u – v**, to be the vector **u + (–v)**. To construct the vector **u – v** we can either

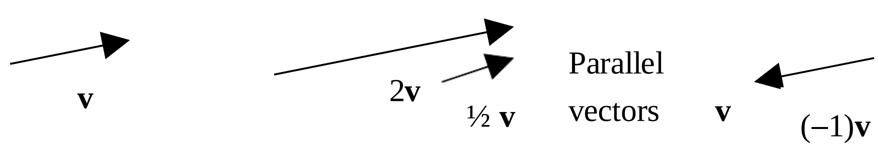
(i) construct the sum of the vector **u** and the vector **–v;** or

(ii) position **u** and **v** so that their initial points coincide; then the vector from the terminal point of **v** to the terminal point of **u** is the vector **u – v**.



Multiplying a vector by a scalar

If **v** is a nonzero vector and *c* is a nonzero scalar, we define the product of *c* and **v**, denoted *c***v**, to be the vector whose length is $|c|$ times the length of **v** and whose direction is the same as that of **v** if $c > 0$ and opposite to that of **v** of $c < 0$. We define $c\mathbf{v} = \mathbf{0}$ if $c = 0$ or if $\mathbf{v} = \mathbf{0}$.



The vectors **v** and *c***v** are **parallel** to each other. Their directions coincide if $c > 0$ and the directions are opposite to each other if $c < 0$. If **u** and **v** are parallel vectors, then there exists a scalar c such that **u** = c**v.** Conversely, if **u** = c**v** and c □ 0, **v** are parallel vectors.