## 2.4 Hubble Expansion Model of the Universe

The **Hubble Expansion Model** of the universe is a foundational concept in cosmology, based on the observation that galaxies are receding from each other in all directions, with their velocity proportional to their distance from us. This discovery is encapsulated in **Hubble's Law**, formulated by American astronomer **Edwin Hubble** in the 1920s. The Hubble Expansion Model provides strong evidence for the **Big Bang Theory** of the universe's origin, which suggests that the universe has been expanding from a hot, dense state for over 13.8 billion years.

### Overview of the Hubble Expansion Model

The Hubble Expansion Model describes how the universe is expanding over time. In this model:

1. **Space itself is stretching**, carrying galaxies with it as it expands.
2. **Galaxies are not moving through space**; instead, the fabric of space between galaxies is growing, causing the galaxies to recede from each other.
3. The rate of this expansion is proportional to the distance between galaxies, a relationship mathematically expressed in **Hubble's Law**.

### Key Components of the Hubble Expansion Model

1. **Hubble's Law**: The central element of the Hubble Expansion Model is **Hubble's Law**, which states that:

$$v = H_0 \times d$$

Where:

- v is the **recessional velocity** of a galaxy (the speed at which it is moving away from the observer).
- $H_0$ is the **Hubble constant**, which measures the rate of expansion of the universe.
- d is the **distance** to the galaxy.

This relationship shows that the farther away a galaxy is, the faster it recedes from us, which directly implies that the universe is expanding. This velocity-distance relationship is often compared to the stretching of a rubber sheet, where dots on the sheet (representing galaxies) move away from each other as the sheet stretches. The expansion is happening at all scales across the universe.

**Cosmological Redshift**:

The **redshift** of light from distant galaxies provides direct evidence for the Hubble Expansion Model. As light travels through space, the wavelength of light is stretched by the expansion of space. This is observed as a shift towards longer wavelengths (red end of the spectrum), known as **cosmological redshift**. The greater the distance to a galaxy, the more significant the redshift.

The relationship between redshift (z) and recessional velocity (v) is given by:

$$v = z \times c$$

Where:

- v is the recessional velocity,
- z is the redshift (defined as the fractional change in the wavelength of light),
- c is the speed of light.

This redshift is a key observational piece of evidence for the expanding universe and is directly tied to Hubble's Law.

**Expanding Space**:

The Hubble Expansion Model suggests that **space itself is stretching**, rather than galaxies merely moving through space. This stretching of space is governed by the equations of general relativity, which describe the behavior of spacetime on cosmic scales.

A key feature of the model is that it does not imply galaxies are moving through space at high speeds; rather, the **space between galaxies is increasing** over time. This phenomenon is often called **metric expansion of space**, and it's a fundamental feature of the universe's evolution.

**Hubble's Constant ($H_0$)**:

The value of **Hubble's constant** is critical to determining the rate of expansion of the universe. It is expressed in units of kilometers per second per megaparsec (km/s/Mpc). A megaparsec (Mpc) is a unit of distance commonly used in cosmology, approximately equal to 3.26 million light-years.

The value of $H_0$ has been measured with increasing accuracy over the years, but there is still some debate regarding its precise value due to differing measurement techniques. Current estimates place the value of $H_0$ between **67.4 km/s/Mpc** and **74.0 km/s/Mpc**.

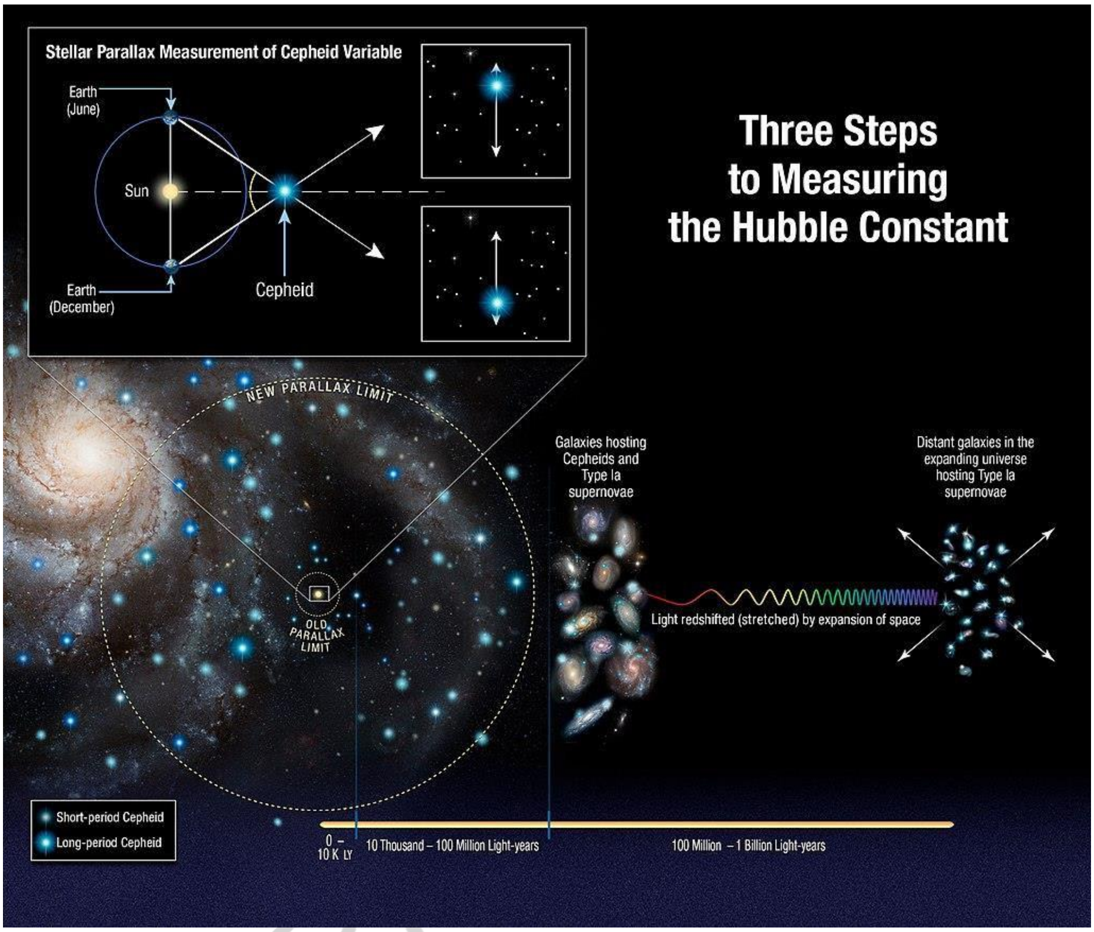


***Fig: Hubble Expansion Model***

## Mathematical Framework Behind the Hubble Expansion Model

The Hubble Expansion Model is rooted in the **Friedmann equations** derived from **general relativity**. These equations describe the expansion of the universe in terms of the **metric of spacetime** and the **energy content of the universe**.

1. **Friedmann-Lemaître-Robertson-Walker (FLRW) Metric**: The FLRW metric describes the geometry of the expanding universe and is based on the assumption that the universe is homogeneous (the same everywhere on large scales) and isotropic (the same in all directions). The equation governing this expansion is:

$$ds^2 = -c^2dt^2 + a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

Where:

- ds is the space-time interval,
- c is the speed of light,
- a(t) is the **scale factor**, which describes how the distance between objects in the universe changes over time,
- k is the spatial curvature constant, which describes the overall geometry of the universe.

**Friedmann Equations**: The Friedmann equations govern the evolution of the scale factor a(t) and the expansion of the universe. These equations are derived from Einstein's field equations of general relativity, which describe how the energy content of the universe influences its expansion.

- The first Friedmann equation, describing the scale factor's evolution, is:

$$\left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2(t)}$$

Where:

- $\dot{a}(t)$ is the time derivative of the scale factor,
- G is the gravitational constant,
- ρ is the energy density of the universe,
- k is the spatial curvature.

The second Friedmann equation governs the acceleration of the universe's expansion:

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3}\left(\rho + 3\frac{p}{c^2}\right)$$

Where:

- p is the pressure
- $\ddot{a}(t)$ is the second time derivative of the scale factor, indicating the acceleration of expansion.

These equations describe how the universe's expansion rate evolves over time and how it is influenced by the different components of the universe's energy budget (matter, radiation, dark energy, etc.).

**Cosmological Implications of Hubble's Law**

1. **The Expanding Universe**: Hubble's discovery that galaxies are receding from each other implies that the universe itself is expanding. This expansion started from a singularity at the **Big Bang**, and it has continued ever since. The more distant a galaxy is, the faster it recedes from us, which means that, on cosmic scales, the universe looks the same no matter where you are.
2. **Cosmic Scale Factor and Age of the Universe**: The expansion of the universe is governed by the scale factor a(t)a(t)a(t), which changes over time. By understanding the current rate of expansion (H0H_0H0) and the amount of time the universe has been expanding, cosmologists can estimate the **age of the universe**. This age is estimated to be approximately **13.8 billion years**.
3. **Cosmic Horizon**: Due to the expansion of space, there is a **cosmic horizon** beyond which objects are receding faster than the speed of light. This means that there are regions of the universe that are beyond our observational reach, and the light from these distant objects will never reach us because the space between is expanding faster than light can travel.

**Challenges and Modifications to Hubble's Model**

While the Hubble Expansion Model has had a profound impact on our understanding of the universe, several key questions remain. One of the biggest challenges in cosmology is determining the **exact value of the Hubble constant**, with different methods yielding slightly different results, leading to a "tension" in current measurements.

Additionally, the discovery of **dark energy** in the late 1990s, through the observation of distant supernovae, has led to modifications of the Hubble Expansion Model. Dark energy is a mysterious force that is causing the acceleration of the universe's expansion, and it represents a new phase of cosmological evolution that was not accounted for in the original model.