1.5 COULOMB'S LAW

Coulomb's Law:

Coulomb's law is an experimental law formulated in 1785 by Charles Augustin de coulomb. It deals with the force a point charge exerts on another point charge.A point charge means a charge that is located on a body whose dimensions are much smaller than other relevant dimensions.

Charge is generally measured in coulomb ($\boldsymbol{C}$).One Coulomb is approximately equivalent to $\mathbf{6 \times 10^{18}}$ **electrons.** It is very large unit charge because one electron charge $\boldsymbol{e = -1.6019 \times 10^{-19}\ C.}$

Coulomb's law states that the force between two very small objects separated by a distance which is large compared to their size is proportional to the charge on each and inversely proportional to the square of the distance between them.

The coulomb's law can be stated that "The force of attraction or repulsion between any two point charges is directly proportional to the product of two charges and inversely proportional to the square of the distance between them."

Consider the two charges $Q_1$and Q_2 separated by a distance $\boldsymbol{r}$.This force of interaction between two point charges is given as follows:

$$\boldsymbol{F \propto \frac{Q_1 Q_2}{r^2}}$$

$$\boldsymbol{F = \frac{Q_1 Q_2}{4\pi\varepsilon r^2}}\ Newtons$$

Where $\boldsymbol{\varepsilon}$ is permittivity of the medium or dielectric constant which is written as

$$\boldsymbol{\varepsilon = \varepsilon_0 \varepsilon_r}\quad Farads/meter$$

Relative permittivity of the medium

$$\boldsymbol{\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}}$$

$$\boldsymbol{\varepsilon_r = \frac{Permittivity\ of\ the\ medium}{Permittivity\ of\ the\ free\ space\ (or) vaccum} = \frac{\varepsilon}{\varepsilon_0}}$$

$$\boldsymbol{Permittivity\ of\ the\ free\ space\ (or) vaccum\ (\varepsilon_0)} = \frac{\mathbf{1}}{\mathbf{36\pi \times 10^9}}$$

$$= \mathbf{8.854} \times \mathbf{10^{-12}}\ \boldsymbol{F/m}$$

Coulom's law in vector form:

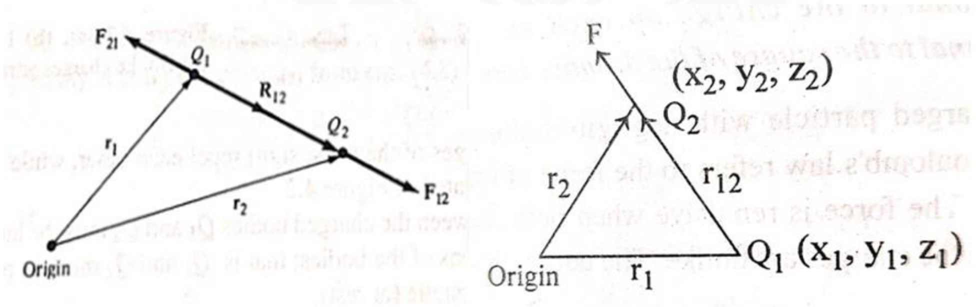

Figure 1.5.1 Coulomb vector force on point charges Q_1 and Q_2

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-107]

$$\boldsymbol{F} = \frac{\boldsymbol{Q_1 Q_2}}{\boldsymbol{4\pi\varepsilon r^2}} \times \boldsymbol{Unit\ vector}$$

Consider two charges $\boldsymbol{Q_1}$and $\boldsymbol{Q_2}$ at a distance vectors $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$ from the origin respectively. $\boldsymbol{r_{12}}$ represents the distant vector form $\boldsymbol{Q_1}$to $\boldsymbol{Q_2}$

$$\boldsymbol{r_{12}} = \boldsymbol{r_2} - \boldsymbol{r_1}$$

The vector $\boldsymbol{F}$ is the force between $\boldsymbol{Q_1}$and $\boldsymbol{Q_2}$

The vector form of coulomb's law is

$$\boldsymbol{F} = \frac{\boldsymbol{Q_1 Q_2}}{\boldsymbol{4\pi\varepsilon r^2}} \times \overrightarrow{\boldsymbol{a_{12}}}$$

The unit vector $$\overrightarrow{\boldsymbol{a_{12}}} = \frac{\boldsymbol{r_2 - r_1}}{|\boldsymbol{r_2 - r_1}|}$$

Substitute $\overrightarrow{\boldsymbol{a_{12}}}$ vector and $\boldsymbol{\varepsilon}$ value in above equation

$$\boldsymbol{F} = \frac{\boldsymbol{Q_1 Q_2}}{\boldsymbol{4\pi\varepsilon_0\varepsilon_r r^2}} \times \overrightarrow{\boldsymbol{a_{12}}}$$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 \varepsilon_r r^2} \times \frac{r_2 - r_1}{|r_2 - r_1|}$$

$\boldsymbol{r}$ is the distance between the charge.

The distance between the charges is $\boldsymbol{r_{12}}$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r_{12}^2} \times \frac{r_2 - r_1}{|r_2 - r_1|}$$

That is $r_{12} = r_2 - r_1$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 (r_2 - r_1)^2} \times \frac{r_2 - r_1}{|r_2 - r_1|}$$

$$r_2 - r_1 = (x_2 - x_1)\overrightarrow{a_x} + (y_2 - y)\overrightarrow{a_y} + (z_2 - z_1)\overrightarrow{a_z}$$

$$(r_2 - r_1)^2 = \left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right]^2$$

$$|r_2 - r_1| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 \left[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2\right]^2} \times \frac{\left[(x_2 - x_1)\overrightarrow{a_x} + (y_2 - y)\overrightarrow{a_y} + (z_2 - z_1)\overrightarrow{a_z}\right]}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 [(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]^2} \times \frac{\left[(x_2 - x_1)\overrightarrow{a_x} + (y_2 - y)\overrightarrow{a_y} + (z_2 - z_1)\overrightarrow{a_z}\right]}{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]^{1/2}}$$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0} \times \frac{\left[(x_2 - x_1)\overrightarrow{a_x} + (y_2 - y)\overrightarrow{a_y} + (z_2 - z_1)\overrightarrow{a_z}\right]}{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]^{5/2}}$$

The force is attractive if the charges are of opposite sign and is repulsive if the charges are alike in sign