**State variability model**

- A modern gigawatt generator with its multistage reheat turbine, including its automatic load frequency control (ALFC) and automatic voltage regulator (AVR) controllers, is characterized by an impressive complexity.

- When all its non- negligibility dynamics are taken into account, including cross-coupling betweencontrol channels, the overall dynamic model may be of the twentieth order.

- The dimensionality barrier can be overcome by means of computer-aided optimal control design methods originated by Kalman. A computer-oriented technique called optimum linear regulator (OLR) design has proven to be particularlyuseful in this regard.

- The OLR design results in a controller that minimizes both transientvariable excursions and control efforts. In terms of power system, this means optimally damped oscillation with minimum wear and tear of control valves.

OLR can be designed using the following steps:

- Casting the system dynamic model in state-variable form and introducing appropriate control forces.

- Choosing an integral-squared-error control index, the minimization of which is the

- control goal.

- Finding the structure of the optimal controller that will minimize the chosencontrol index.

Dynamic State Variable Model

- The LFC methods discussed so far are not entirely satisfactory. In order to have more satisfactory control methods, optimal control theory has to be used. For this purpose, the power system model must be in a state variable model
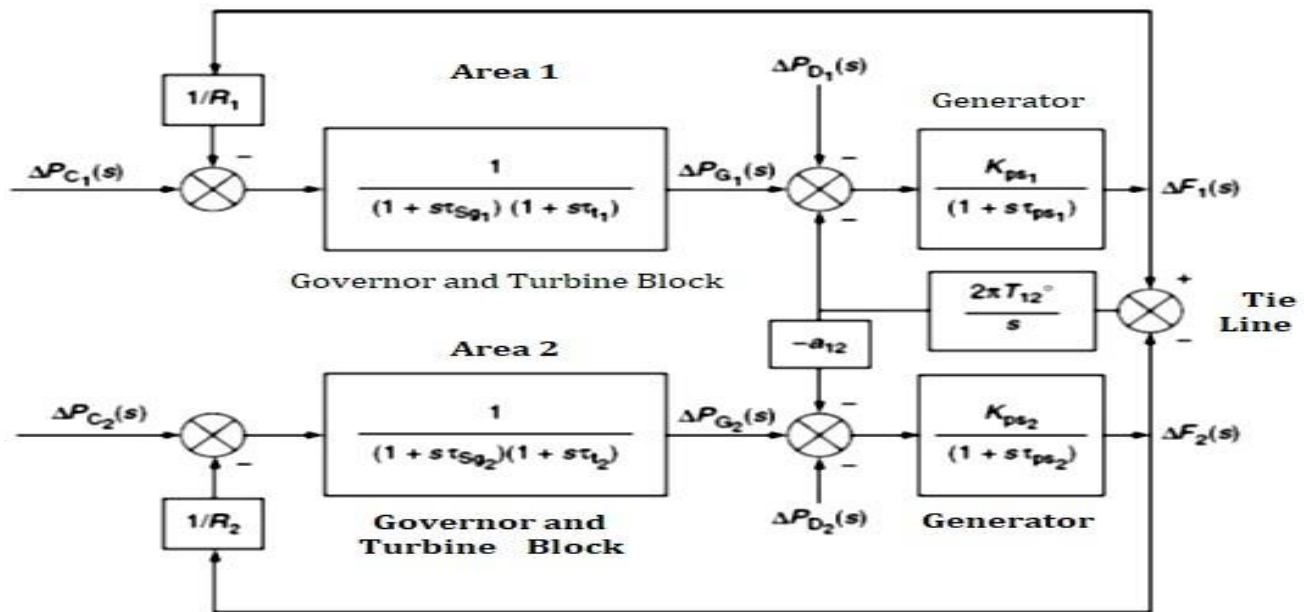
From the block diagram write the 's domain' equations

$$\Delta F_1(s) = \frac{K_{ps1}}{1 + s\tau_{ps1}} \left[\Delta P_{g1}(s) - \Delta P_{d1}(s) - \Delta P_{TL1}(s)\right]$$

$$\Delta F_2(s) = \frac{K_{ps2}}{1 + s\tau_{ps2}} \left[\Delta P_{g2}(s) - \Delta P_{d2}(s) - \Delta P_{TL2}(s)\right]$$

$$\Delta X_{E1}(s) = \frac{1}{1 + s\tau_{sg1}} \left[\Delta P_{C1}(s) - F_1(s)/R_1\right]$$

$$\Delta X_{E2}(s) = \frac{1}{1 + s\tau_{sg2}} \left[\Delta P_{C2}(s) - F_2(s)/R_2\right]$$

$$\Delta P_{G1}(s) = \frac{1}{1 + s\tau_{t1}} \left[\Delta X_{E1}(s)\right]$$

$$\Delta P_{G2}(s) = \frac{1}{1 + s\tau_{t2}} \left[\Delta X_{E2}(s)\right]$$

$$\Delta P_{TL1}(s) = \frac{2\pi\, T_{12}}{s} \left[\Delta F_1(s) - \Delta F_2(s)\right]$$

Where XE1 (s) and XE2 (s) are the Laplace transforms of the movements of the main positions in the speed governing mechanism of the two areas.

By taking inverse Laplace transform for the above equations, we get a set of seven differential equations. These are the time-domain equations, which describe the small-disturbance dynamic behavior of the power system.

$$(1 + s\tau_{ps1}) \Delta F_1(s) = K_{ps1}[\Delta P_{g1}(s) - \Delta P_{d1}(s) - \Delta P_{TL1}(s)]$$

$$s\tau_{ps1} \Delta F_1(s) = -\Delta F_1(s) + K_{ps1}[\Delta P_{g1}(s) - \Delta P_{d1}(s) - \Delta P_{TL1}(s)]$$

$$s\Delta F_1(s) = \frac{1}{\tau_{ps1}}\{-\Delta F_1(s) + K_{ps1}[\Delta P_{g1}(s) - \Delta P_{d1}(s) - \Delta P_{TL1}(s)]\}$$

Taking the inverse Laplace transform of the above equation, we get

$$\frac{d}{dt}[\Delta F_1] = \frac{1}{\tau_{ps1}}\{-\Delta F_1 + K_{ps1}\Delta P_{g1} - K_{ps1}\Delta P_{d1} - K_{ps1}\Delta P_{TL1}\}$$

In a similar way, the remaining equations can be rearranged and an inverse Laplace transform is found. Then, the entire set of differential equations is

$$\frac{d}{dt}[\Delta F_2] = \frac{1}{\tau_{ps2}}\{-\Delta F_2 + K_{ps2}\Delta P_{g2} - K_{ps2}\Delta P_{d2} - K_{ps2}\Delta P_{TL1}a_{12}\}$$

$$\frac{d}{dt}(\Delta X_{E1}) = \frac{1}{\tau_{sg1}}[-\Delta X_{E1} + \Delta P_{C1} - \Delta F_1/R_1]$$

$$\frac{d}{dt}(\Delta X_{E2}) = \frac{1}{\tau_{sg2}}[-\Delta X_{E2} + \Delta P_{C2} - \Delta F_2/R_2]$$

$$\frac{d}{dt}(\Delta P_{G1}) = \frac{1}{\tau_{t1}}[-\Delta P_{G1} + \Delta X_{E1}]$$

$$\frac{d}{dt}(\Delta P_{G2}) = \frac{1}{\tau_{t2}}[-\Delta P_{G2} + \Delta X_{E2}]$$

$$\frac{d}{dt}(\Delta P_{TL1}) = 2\pi T_{12}[\Delta F_1 - \Delta F_2]$$

The state variables are a minimum number of those variables, which contain sufficient information about the past history with which all future states of the system can be determined for known control inputs. For the two area system under consideration, the state variables would be Δf1, Δf2 , ΔXE1 , ΔXE2 , ΔPsg1 , ΔPsg2 and ΔPTL1 ; seven in number. Denoting the above variables by x1, x2, x3, x4, x5, x6, and x7 and arranging them in a column vector as

$$\overline{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} = \begin{bmatrix} \Delta F_1 \\ \Delta F_2 \\ \Delta X_{E1} \\ \Delta X_{E2} \\ \Delta P_{sG1} \\ \Delta P_{sG2} \\ \Delta P_{TL1} \end{bmatrix}$$

Where X is called a state vector

The control variables ΔPc1 and ΔPc2 are denoted by the symbols u1 and u2, respectively, as

$$\overline{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \equiv \begin{bmatrix} \Delta P_{c1} \\ \Delta P_{c2} \end{bmatrix}$$

where u is called the control vector

The disturbance variables ΔPD1 and ΔPD2, since they create perturbations in the system, are denoted by p1 and p2, respectively, as

$$\overline{p} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \equiv \begin{bmatrix} \Delta P_{D1} \\ \Delta P_{D2} \end{bmatrix}$$

where P is called the disturbance vector

The above state equations can be written in a matrix form as

$$
\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \\ \dot{x_4} \\ \dot{x_5} \\ \dot{x_6} \\ \dot{x_6} \end{bmatrix} =
\begin{bmatrix}
\dfrac{1}{\tau_{ps1}} & 0 & 0 & 0 & \dfrac{K_{ps1}}{\tau_{ps1}} & 0 & -\dfrac{K_{ps1}}{\tau_{ps1}} \\[2mm]
0 & -\dfrac{1}{\tau_{ps2}} & 0 & 0 & 0 & \dfrac{K_{ps2}}{\tau_{ps2}} & -\dfrac{K_{ps1}}{\tau_{ps1}} \\[2mm]
-\dfrac{1}{R_1\tau_{sg1}} & 0 & -\dfrac{1}{\tau_{sg1}} & 0 & 0 & 0 & 0 \\[2mm]
0 & -\dfrac{1}{R_2\tau_{sg2}} & 0 & -\dfrac{1}{\tau_{sg2}} & 0 & 0 & 0 \\[2mm]
0 & 0 & -\dfrac{1}{\tau_{t1}} & 0 & -\dfrac{1}{\tau_{t1}} & 0 & 0 \\[2mm]
0 & 0 & 0 & \dfrac{1}{\tau_{t2}} & 0 & -\dfrac{1}{\tau_{t2}} & 0 \\[2mm]
2\pi T_{12} & -2\pi T_{12} & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}
$$

$$
+ \begin{bmatrix}
0 & 0 \\
0 & 0 \\
\dfrac{1}{\tau_{sg1}} & 0 \\[2mm]
0 & \dfrac{1}{\tau_{sg2}} \\[2mm]
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}
+ \begin{bmatrix}
-\dfrac{K_{ps1}}{\tau_{ps1}} & 0 \\[2mm]
0 & -\dfrac{K_{ps1}}{\tau_{ps1}} \\[2mm]
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \quad (1)
$$

In the present case, their dimensions are (7×7), (7×2), and (7×2), respectively. Equation (2) is a shorthand form of Equation (1), and Equation (1) constitutes the dynamic 'state-variable model' of the considered two-area system.

The differential equations can be put in the above form only if they are linear. If the differential equations are non-linear, then they can be expressed in the more general form as

$$\bar{\dot{X}} = f(\bar{x}, \bar{u}, \bar{p})$$