MAJOR AND MINOR LOSSES OF FLOW IN PIPES

Major Losses

The major losses of energy are due to friction. Which are considerable hence it is called as major losses. It is determined by Darcy- Weisbach formula and Chezy's formula. Head loss due to friction is denoted by h_f.

Darcy- Weisbach formula

$$h_f = \frac{4 f L V^2}{2 g d}$$

Where, hf – loss of head due to friction in meter of fluid

f - Coefficient of friction

Coefficient of friction is function of Reynolds's Number (Re).

If Re is less than 2000 (i.e. laminar flow) $f = \frac{16}{Re}$

If Re is greater than 4000 (i.e. turbulent flow) $f = \frac{0.0719}{Re^{1/4}}$ L- Length of pipe in m.

V- Velocity of flow in m/s.

d- Diameter of pipe in m.

Let, $V = \frac{Q}{A}$, Hence Darcy-Weisbach formula in the term of discharge Q,

$$h_f = \frac{f L Q^2}{12 d^5}$$

Chezy's formula-

$$V = C\sqrt{mi}$$

Where V – velocity of flow in m/s

C – Chezy's constant

i – Loss of head per unit length of pipe = $\frac{hf}{L}$

m – Hydraulic mean depth = (Area of flow/ Perimeter) = A / P

$m = \frac{d}{4}$ for pipe flow

Minor losses

The losses due to disturbances in flow pattern or due to change in velocity are called as minor losses. These losses may occur due to sudden change in the area of flow and the direction of flow. These losses are less as compare to major losses. The minor loss of the head (energy) includes the following cases:

1. Loss of head due to sudden enlargement

2. Loss of head due to sudden contraction

3. Loss of head at the entrance of a pipe

4. Loss of head at the exit of a pipe

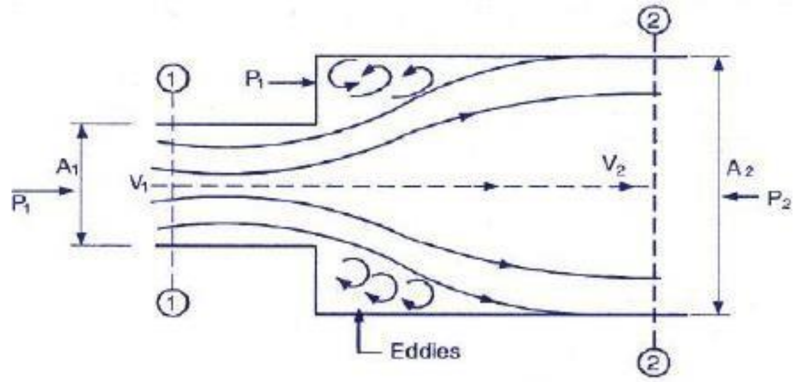
5. Loss of head due to bends

6. Loss of head in various pipe fittings

7. Loss of head due to obstruction

1. Loss of head due to sudden enlargement

Fig. represents a pipe in which fluid experiences sudden enlargement. Here the head lossoccurs due to the separation of the flow at the periphery of the smaller pipe, which leads to eddying motion in the corner region.



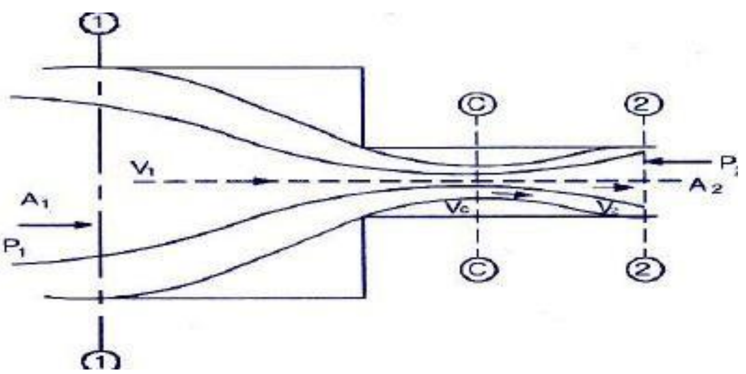
The Equation gives head loss due to sudden expansion.

$$he = \frac{(V_1 - V_2)^2}{2g}$$

Where, V1 = Velocity of fluid at section 1-1
V2 = Velocity of fluid at section 2-2

2. Loss of Head Due to Sudden Contraction

Fig. represents a pipe in which fluid experiences sudden contraction. The stream lines are converging from section 1-1 to section C-C. The head loss occurs only after the vena contracta CC. This is because the flow up to this section is accelerating and the boundary layer separation does not occur.



Using Bernoulli's equation, continuity and momentum equation at section 1-1 and 2-2, it can be proved that head loss due to sudden contraction is,

$$hc = \left(\frac{1}{c_c} - 1\right)^2 \frac{v_2^2}{2\,g}$$

Where, V2 = Velocity of fluid at section 2-2

Cc = Coefficient of contraction = Ac /A2

If Cc not given,

$$hc = 0.5\,\frac{v_2^2}{2\,g}$$

3. Loss of Head at the entrance of a pipe

The loss of head at the entrance of pipe is a similar case to loss of head due to sudden contraction as there is an abrupt reduction in area from an area of reservoir to area of a pipe. The loss of head is caused mainly by the turbulence created by the sudden enlargement of the jet after it has passed through the vena contracta.

$$h = 0.5\,\frac{v^2}{2\,g}$$

4. Loss of head at the exit of a pipe

When the fluid from the pipe enters into a relatively larger reservoir the entire velocity is dissipated. If V is the velocity of fluid in a pipe, the head loss at exit is given by

$$h = \frac{v^2}{2\,g}$$

5. Loss of head due to bends

The loss of head in bends provided in pipes may be expressed as,

$$h = k\,\frac{v^2}{2\,g}$$

V is the mean velocity of flow of liquid and K = coefficient of bend and is depends on the angle of bend, radius of the curvature and diameter of pipe.

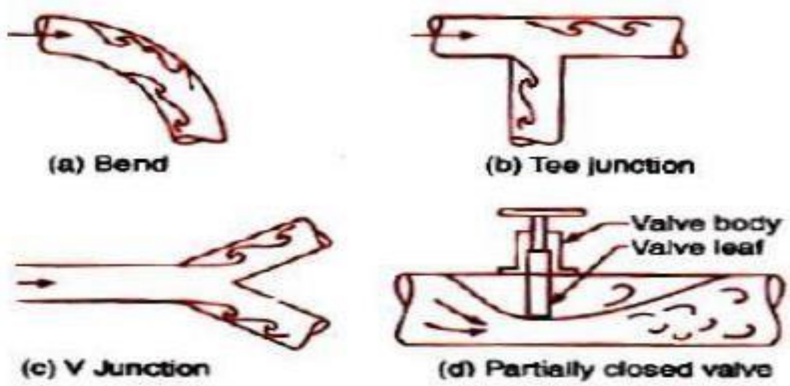
6. Loss of Head in Various Pipe Fittings

Pipe fittings in a piping system cause obstruction to flow and the loss of head occurs. The loss of head may be expressed as,

$$h = k\,\frac{v^2}{2g}$$

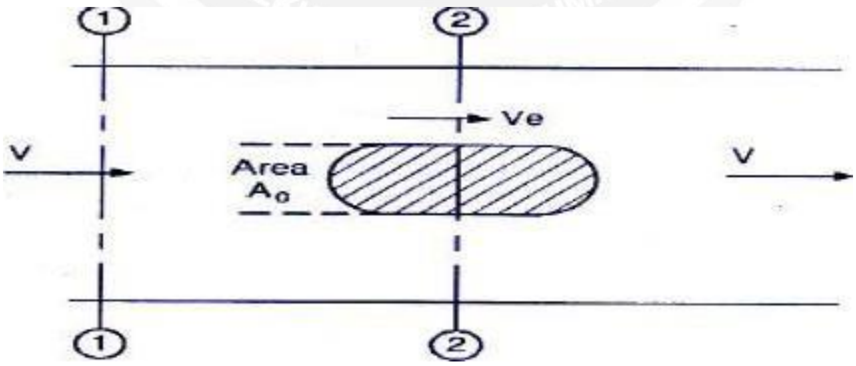
Where, K = Coefficient of pipe fitting

Various pipe fitting shown in following figure,



7. Loss of head due to obstruction

The loss of head due to obstruction in a pipe takes place due to reduction in the cross sectional area of the pipe by the presence of obstruction which is followed by an abrupt enlargement of the stream lines beyond the obstruction. (Shown in figure)



Let, V = Velocity of fluid in pipe

A0 = Maximum area of obstruction

A = Area of pipe

ho = head loss due to obstruction

$$h_o = \left[\frac{1}{Cc\,(A-Ao)} - 1\right]^2 \frac{V^2}{2g}$$

PROBLEM 1. At a sudden enlargement of a water main from 240mm to 480mm diameter, the hydraulic gradient rises by 10mm. Estimate the rate of flow.

Given: Dia. of smaller pipe D1= 240mm =0.24m

$$\text{Area } A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.24)^2$$

Dia. of larger pipe D2 = 480mm = 0.48m

$$\text{Area } A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4}(0.48)^2$$

Rise of hydraulic gradient i.e. $Z_2 + \frac{P_2}{\rho g} - Z_1 + \frac{P_1}{\rho g} = 10\text{mm} = \frac{10}{1000} m = \frac{1}{100} m$

Let the rate of flow = Q

Applying Bernoulli's equation to both sections i.e smaller and larger sections

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \text{Head loss due to enlargement} \quad \text{(1)}$$

But head loss due to enlargement, $h_e = \frac{(V_1 - V_2)^2}{2g}$ ———(2)

From continuity equation, we have $A_1 V_1 = A_2 V_2$ $\quad V_1 = \frac{A_2 V_2}{V_1}$

$$V_1 = \frac{\frac{\pi}{4} D_2^2 V_2}{\frac{\pi}{4} D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{0.48}{0.24}\right)^2 V_2 = 2^2 V_2 = 4V_2$$

Substituting this value in equation (2), we get

$$h_e = \frac{(4V_2 - V_2)^2}{2g} = \frac{(3V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

Now substituting the value of he and V1 in equation (1)

$$\frac{P_1}{\rho g} + \frac{(4V_2)^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \frac{9V_2^2}{2g}$$

$$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \frac{P_2}{\rho g} + Z_2 - \frac{P_1}{\rho g} + Z_1$$

But Hydraulic gradient rise = $\frac{P_2}{\rho g} + Z_2 - \frac{P_1}{\rho g} + Z_1 = \frac{1}{100} m$

$\frac{6V_2^2}{2g} = \frac{1}{100}$ m $\qquad V_2 = \frac{2\times9.81}{6\times100} = 0.1808 = 0.181 m/sec$

Discharge $\quad Q = A_2 V_2 = \frac{\pi}{4} D_2^2 V_2$

$$= \frac{\pi}{4}(0.48)^2 \times 0.181 = 0.03275 m^3/sec$$

$= \underline{32.75 Lts/sec}$

PROBLEM 2. A 150mm dia. pipe reduces in dia. abruptly to 100mm dia. If the pipe carries water at 30lts/sec, calculate the pressure loss across the contraction. Take co-efficient of contraction as 0.6.

Given: Dia. of larger pipe D1 = 150mm = 0.15m

Area of larger pipe $A_1 = \frac{\pi}{4}(0.15)^2 = 0.01767 m^2$

Dia. of smaller pipe D2 = 100mm = 0.10m

Area of smaller pipe $A_2 = \frac{\pi}{4}(0.10)^2 = 0.007854 m^2$

Discharge Q = 30 lts/sec = 0.03m3/sec Co-efficient of contraction CC = 0.6

From continuity equation, we have Q = A1 V1 = A2 V2

$$V_1 = \frac{Q}{A_1} = \frac{0.03}{0.01767} = 1.697\ m/sec$$

$$V_2 = \frac{Q}{A_2} = \frac{0.03}{0.007854} = 3.82\ m/sec$$

Applying Bernoulli‟s equation before and after contraction

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_c \qquad \text{\_\_\_\_\_\_\_\_\_\_(1)}$$

But Z1 = Z2 and hc the head loss due to contraction is given by the equation

$$hc = \left(\frac{1}{C_c} - 1\right)^2 \frac{v_2^2}{2g} = \frac{3.82^2}{2\times9.81}\left(\frac{1}{0.6} - 1\right)^2 = 0.33$$

Substituting these values in equation (1), we get

$$\frac{P_1}{\rho g} + \frac{1.697^2}{2\times9.81} = \frac{P_2}{\rho g} + \frac{3.82^2}{2\times9.81} + 0.33$$

$$\frac{P_1}{\rho g} + 0.1467 = \frac{P_2}{\rho g} + 0.7438 + 0.33$$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = 0.7438 + 0.33 - 0.1467 = 0.9271\ m\ of\ Water$$

$$P_1 - P_2 = \rho g \times 0.9271 = 1000 \times 9.81 \times 0.9271 = 0.909 \times 10^4\ N/m^2$$

$$= 0.909\ N/cm^2$$

Pressure loss across contraction = P1- P2 = 0.909N/cm^2

PROBLEM 3. Water is flowing through a horizontal pipe of diameter 200mm at a velocity of 3m/sec. A circular solid plate of diameter150mm is placed in the pipe to obstruct the flow. Find the loss of head due to obstruction in the pipe, if C_C =0.62.

Given: Diameter of pipe D = 200mm =

0.2mVelocity V = 3m/sec

$$\text{Area of pipe } A = \frac{\pi}{4} D^2 = \frac{\pi}{4}(0.2)^2 = 0.03141 m^2$$

Diameter of obstruction d = 150mm =0.15m

$$\text{Area of obstruction } a = \frac{\pi}{4}(0.15)^2 = 0.01767 m$$

C_C = 0.62

The head loss due to obstruction $h_o = \left[\frac{1}{Cc\ (A-Ao)} - 1\right]^2 \frac{V^2}{2\,g}$

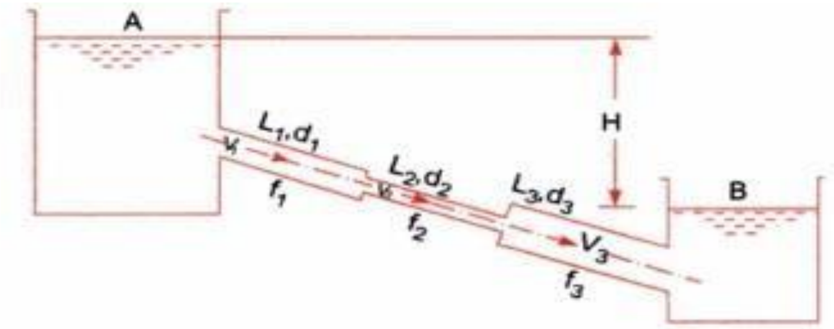
$$= \frac{9}{19.62}\ [3.687 - 1]^2$$

= **3.311m**

6 PIPES IN SERIES AND IN PARALLEL

PIPES IN SERIES:

When pipes of different lengths and different diameters are connected end to end to form a pipe line, such arrangement or connection of pipes will be considered as pipes in series or compound pipes. Following figure, displayed here, indicates the arrangement of connection of three pipes in series.



Let us consider the following terms from above figure

L_1, L_2 and L_3: Length of pipes 1, 2 and 3 respectively d_1, d_2 and d_3: Diameter of pipes 1, 2 and 3 respectively V_1, V_2 and V_3: Velocity of flow through pipes 1, 2 and 3 respectively
f_1, f_2 and f_3: Co-efficient of friction for pipes 1, 2 and 3 respectively
H = Difference of water level in two tanks

We must note it here that difference in liquid surface level will be equal to the sum of total head loss in the pipes.

If we neglect the minor head losses, we will have following equation for total head loss as mentioned here.

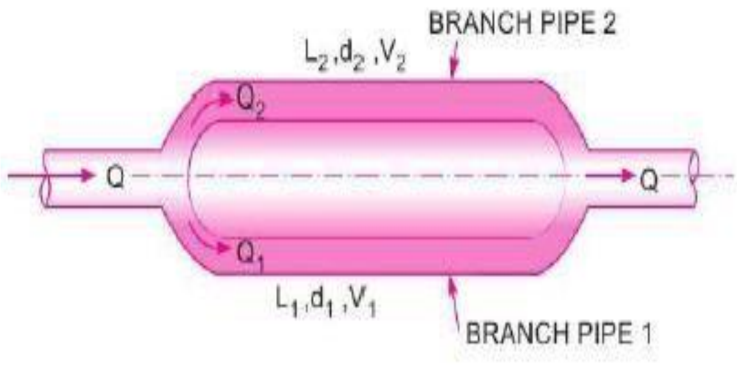
$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}$$

Let us consider that co-efficient of friction i.e. f is same for all three pipes and therefore we can write the equation for head loss as mentioned here.

$$H = \frac{4fL_1V_1^2}{d_1 \times 2g} + \frac{4fL_2V_2^2}{d_2 \times 2g} + \frac{4fL_3V_3^2}{d_3 \times 2g}$$

$$= \frac{4f}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\right]$$

PIPES IN PARALLEL:



When a main pipeline divides in to two or more parallel pipes, which may again join together downstream and continue as main line, the pipes are said to be in parallel. The pipes are connected in parallel in order to increase the discharge passing through the main.

It is analogous to parallel electric current in which the drop in potential and flow of electric current can be compared to head loss and rate of discharge in a fluid flow respectively.

The rate of discharge in the main line is equal to the sum of the discharges in each of the parallel pipes.

Thus Q = Q1 + Q2

The flow of liquid in pipes (1) and (2) takes place under the difference of head between the sections A and B and hence the loss of head between the sections A and B will be the same whether the liquid flows through pipe (1) or pipe (2). Thus if D_1, D_2 and L_1, L_2 are the diameters and lengths of the pipes (1) and (2) respectively, then the velocities of flow V_1 and V_2 in the two pipes must be such as to give

$$\frac{4 f_1 L_1 V_1^2}{d_1 \times 2g} = \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g}$$

$$f_1 = f_2, \text{ then } \frac{L_1 V_1^2}{d_1 \times 2g} = \frac{L_2 V_2^2}{d_2 \times 2g}$$

EQUIVALENT PIPE

In practice adopting pipes in series may not be feasible due to the fact that they may be of unistandard size (ie. May not be comemercially available) and they experience other minor losses. Hence, the entire system will be replaced by a single pipe of uniform diameter D, but of the same length $L = L_1 + L_2 + L_3$ such that the head loss due to friction for both the pipes, viz equivalent pipe & the compound pipe are the same.

For a compound pipe or pipes in series

$$h_f = hf_1 + hf_2 + hf_3$$

$$h_f = \frac{8 f L_1 Q^2}{g \pi^2 D_1^5} + \frac{8 f L_2 Q^2}{g \pi^2 D_2^5} + \frac{8 f L_3 Q^2}{g \pi^2 D_3^5} --- (1)$$

for an equivalent pipe $h_f = \frac{8 fLQ^2}{g\pi^2 D^5} --- (2)$

Equating (1) & (2) and simplifying $\frac{L}{D^5} = \frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}$

$$\text{or} \quad D = \left\{ \frac{L}{\frac{L_1}{D_1^5} + \frac{L_2}{D_2^5} + \frac{L_3}{D_3^5}} \right\}^{\frac{1}{5}}$$

PROBLEM 1:The difference in water surface levels in two tanks, which are connected by three pipes in series of lengths 400 m, 200 m and 300 m and of diameters 400 mm, 300 mm and 200 mm respectively, is 16m. Estimate the rate of flow of water if co-efficient of friction for these pipes is same and equal to 0.005, considering: (i) minor losses also (ii) neglecting minor losses.

Solution. Given :

Difference of water levels, $H = 16$ m

Length and dia. of pipe 1, $L_1 = 400$ m and $d_1 = 400$ mm $= 0.4$ m

Length and dia. of pipe 2, $L_2 = 200$ m and $d_2 = 200$ mm $= 0.2$ m

Length and dia. of pipe 3, $L_3 = 300$ m and $d_3 = 300$ mm $= 0.3$ m

Also $f_1 = f_2 = f_3 = 0.005$

(*i*) **Discharge through the compound pipe first neglecting minor losses.**

Let V_1, V_2 and V_3 are the velocities in the 1st, 2nd and 3rd pipe respectively.

From continuity, we have $A_1V_1 = A_2V_2 = A_3V_3$

$$V_2 = \frac{A_1V_1}{A_2} = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_2^2} \times V_1 = \frac{d_1^2}{d_2^2} V_1 = \left(\frac{0.4}{0.2}\right)^2 V_1 = 4V_1$$

$$V_3 = \frac{A_1V_1}{A_3} = \frac{\frac{\pi}{4}d_1^2}{\frac{\pi}{4}d_3^2} \times V_1 = \frac{d_1^2}{d_3^2} V_1 = \left(\frac{0.4}{0.2}\right)^2 V_1 = 1.77V_1$$

$$H = \frac{4f_1L_1V_1^2}{d_1 \times 2g} + \frac{4f_2L_2V_2^2}{d_2 \times 2g} + \frac{4f_3L_3V_3^2}{d_3 \times 2g}$$

$$16 = \frac{4 \times 0.005 \times 400 \times V_1^2}{0.4 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 200 \times (4V_1)^2}{0.2 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 300}{0.3 \times 2 \times 9.81} \times (1.77\ V_1)^2$$

$$= \frac{V_1^2}{2 \times 9.81}\left(\frac{4 \times 0.005 \times 400}{0.4} + \frac{4 \times 0.005 \times 200 \times 16}{0.2} + \frac{4 \times 0.005 \times 300 \times 3.157}{0.3}\right)$$

$$16 = \frac{V_1^2}{2 \times 9.81}(20 + 320 + 63.14) = \frac{V_1^2}{2 \times 9.81} \times 403.14$$

$$\therefore \quad V_1 = \sqrt{\frac{16 \times 2 \times 9.81}{403.14}} = 0.882 \text{ m/s}$$

$\therefore$ Discharge, $\quad Q = A_1 \times V_1 = \frac{\pi}{4}(0.4)^2 \times 0.882 = \mathbf{0.1108\ m^3/s.}$

(*ii*) **Discharge through the compound pipe considering minor losses also.**
Minor losses are :

(*a*) At inlet, $\quad h_i = \frac{0.5\,V_1^2}{2g}$

(*b*) Between 1st pipe and 2nd pipe, due to contraction,

$$h_c = \frac{0.5\,V_2^2}{2g} = \frac{0.5\left(4V_1^2\right)}{2g} \qquad (\because V_2 = 4V_1)$$

$$= \frac{0.5 \times 16 \times V_1^2}{2g} = 8 \times \frac{V_1^2}{2g}$$

(*c*) Between 2nd pipe and 3rd pipe, due to sudden enlargement,

$$h_e = \frac{\left(V_2 - V_3\right)^2}{2g} = \frac{(4V_1 - 1.77V_1)^2}{2g} \qquad (\because V_3 = 1.77\,V_1)$$

$$= (2.23)^2 \times \frac{V_1^2}{2g} = 4.973\,\frac{V_1^2}{2g}$$

(*d*) At the outlet of 3rd pipe, $h_o = \frac{V_3^2}{2g} = \frac{(1.77V_1)^2}{2g} = 1.77^2 \times \frac{V_1^2}{2g} = 3.1329\,\frac{V_1^2}{2g}$

The major losses are $\quad = \frac{4f_1 \times L_1 \times V_1^2}{d_1 \times 2g} + \frac{4f_2 \times L_2 \times V_2^2}{d_2 \times 2g} + \frac{4f_3 \times L_3 \times V_3^2}{d_3 \times 2g}$

$$= \frac{4 \times 0.005 \times 400 \times V_1^2}{0.4 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 200 \times (4V_1)^2}{0.2 \times 2 \times 9.81} + \frac{4 \times 0.005 \times 300 \times (1.77V_1)^2}{0.3 \times 2 \times 9.81}$$

$$= 403.14 \times \frac{V_1^2}{2 \times 9.81}$$

$\therefore$ Sum of minor losses and major losses

$$= \left[\frac{0.5\,V_1^2}{2g} + 8 \times \frac{V_1^2}{2g} + 4.973\frac{V_1^2}{2g} + 3.1329\frac{V_1^2}{2g}\right] + 403.14\,\frac{V_1^2}{2g}$$

$$= 419.746 \frac{V_1^2}{2g}$$

But total loss must be equal to H (or 16 m)

$$\therefore \quad 419.746 \times \frac{V_1^2}{2g} = 16 \quad \therefore \quad V_1 = \sqrt{\frac{16 \times 2 \times 9.81}{419.746}} = 0.864 \text{ m/s}$$

$$\therefore \text{ Discharge,} \quad Q = A_1 V_1 = \frac{\pi}{4} (0.4)^2 \times 0.864 = \mathbf{0.1085 \ m^3/s.}$$

PROBLEM 2: Three pipes of length 800m,500m and 400m of diameter 500mm,400mm and 300mm respectively are connected are connected in series these pipes are to be replaced by a single pipe of length 1700m.find the diameter of single pipe.

Solution. Given :

Length of pipe 1, $L_1 = 800$ m and dia., $d_1 = 500$ mm $= 0.5$ m

Length of pipe 2, $L_2 = 500$ m and dia., $d_2 = 400$ mm $= 0.4$ m

Length of pipe 3, $L_3 = 400$ m and dia., $d_3 = 300$ mm $= 0.3$ m

Length of single pipe, $L = 1700$ m

Let the diameter of equivalent single pipe $= d$

$$\frac{L}{d^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}$$

$$\frac{1700}{d^5} = \frac{800}{.5^5} + \frac{500}{.4^5} + \frac{400}{0.3^5}$$

$$= 25600 + 48828.125 + 164609 = 239037$$

$$\therefore \quad d^5 = \frac{1700}{239037} = .007118$$

$$\therefore \quad d = (.007188)^{0.2} = 0.3718 = \mathbf{371.8 \ mm.}$$