4.2 WEDGE FRICTION

Problems involving friction can be broadly classified into two types.

• In the first type of problems all forces acting on the object and coefficients of friction μ_s and μ_k are known and we have to determine whether the object will move or not.

In such problems, the frictional force is also unknown and has to be determined.

• The following procedure can be adopted for such problems:

1) Draw F.B.D. of object without frictional force.

2) Choose x-axis parallel to the direction in which object can move and y-axis perpendicular to it.

3) Use $\sum F_y = 0$ to find normal reaction N.

4) Find limiting static friction force $(F_r)_{max} = \mu_s N$

5) Find $\sum F_x$, excluding frictional force. This will be net force trying to move the object.

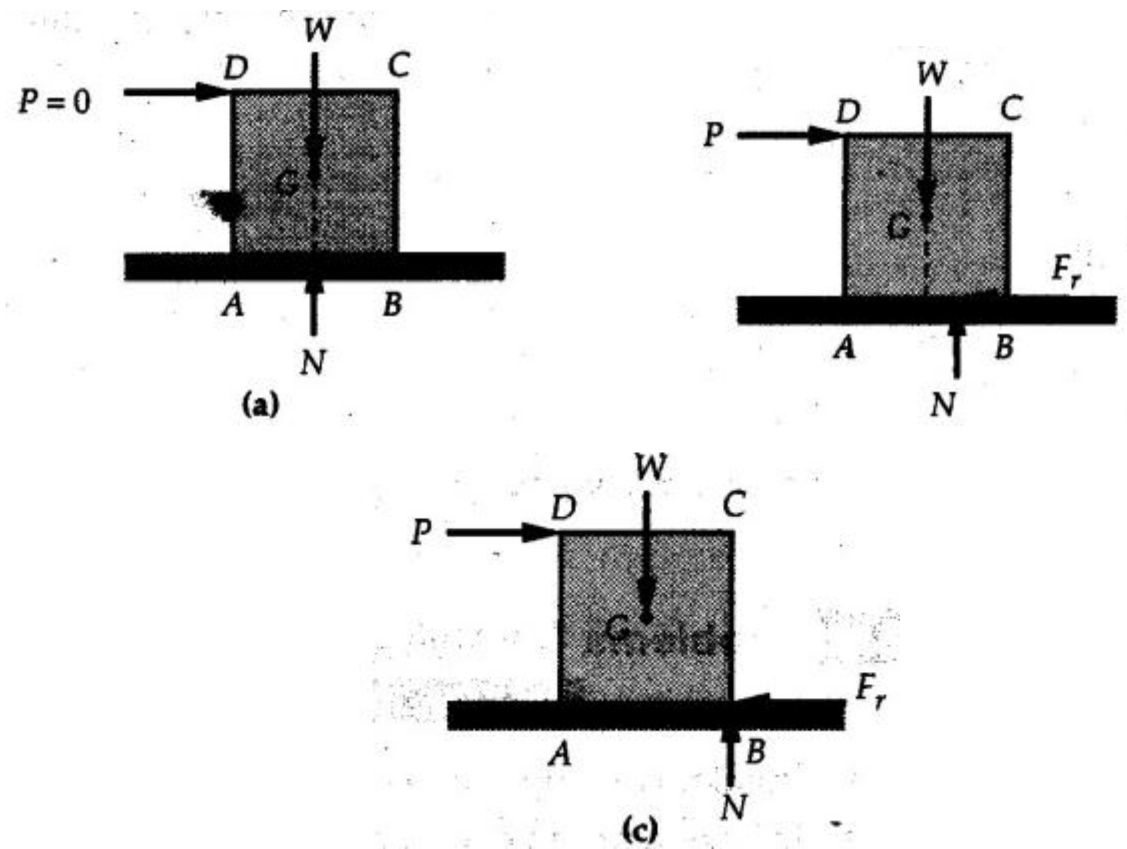
6) If $\left|\sum F_x\right| \leq (F_r)_{max}$, object does not slide. In such a case, the actual value of frictional force will have same magnitude as $\sum F_x$ but opposite direction.

i.e. $F_r = \sum F_x$ in opposite direction.

7) If $\left|\sum F_x\right| > (F_r)_{max}$, object slides and $F_r = \mu_k N$ in a direction opposite to ΣF_x.

• In problems where dimensions of object are given, the object may either slide or overturn.

• Consider an object as shown in Fig. 8.6.1 (a), where applied force P is zero.



The normal reaction N has same line of action as weight W but opposite direction.

• There will be no frictional force.

• When applied force P is increased, the point of application of N shifts towards right as shown in Fig. (b).

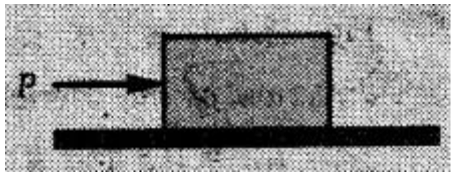
• If P is increased further, when the object is just about to overturn, N will act at the right end B as shown in Fig. (c).

• We can find the value of force P required to overturn the object by taking moment about the right edge B of the object.

• In the second type of problems, it is known that there is impending motion. In such problems draw F.B.D., use F_r = use $F_r = \mu_s$ N and use condition for equilibrium.

Solved Examples

1.Determine whether the 50 kg block shown in Fig. is in equilibrium and find the magnitude and direction of the frictional force when a) P = 100 N and b) P 200 N. Take $\mu_s = 0.3$ and $\mu_k = 0.2$.



Solution:

The F.B.D. of block is shown in Fig. (a) excluding frictional force

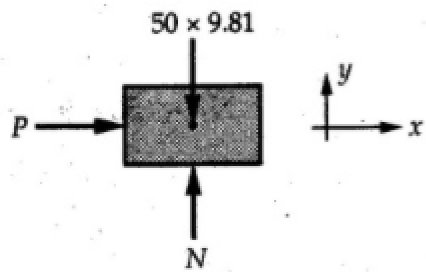


Fig. (a)

$$\sum F_y = 0$$

$$N - 50 \times 9.81 = 0$$

$$\therefore \quad N = 490.5 \text{ N}$$

$$(F_r)_{max} = \mu_s N$$

$$= 0.3 \times 490.5$$

$$\therefore \quad (F_r)_{max} = 147.15 \text{ N}$$

$$\sum F_x = P\ (\rightarrow)$$

a) For $P = 100$ N,

$$\sum F_x = 100 \text{ N}\ (\rightarrow)$$

$$\therefore \quad \sum F_x < (F_r)_{max}$$

∴ Block is in equilibrium

$$\therefore \quad F_r = \sum F_x \text{ in opposite direction}$$

$$\therefore \quad \boxed{F_r = 100 \text{ N} \leftarrow}$$

b) For $P = 200\text{ N}$

$$\sum F_x = 200\text{ N} \rightarrow$$

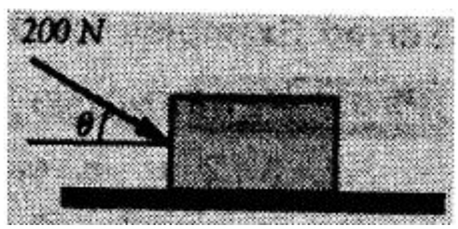
$$\sum F_x > (F_r)_{max}$$

$\therefore$ Object moves towards right (in the direction of $\sum F_x$) and

$$F_r = \mu_k N = 0.2 \times 490.5$$

$$\therefore \quad \boxed{F_r = 98.1\text{ N} \leftarrow}$$

2.Determine whether the 50 kg block shown in Fig. is in equilibrium and find the magnitude and direction of the frictional force when a) θ = 10° and b) θ = 40°. Take μ_s = 0.3 and μ_k = 0.2.



Solution:

The F.B.D. of block is shown in Fig. (a) without frictional force.

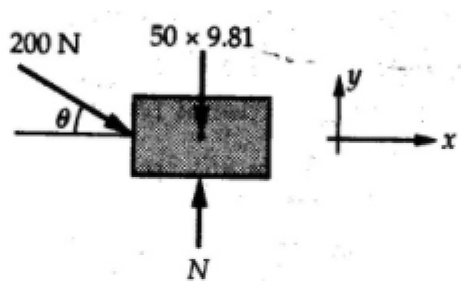


Fig (a)

$$\sum F_y = 0$$

$$N - 50 \times 9.81 - 200 \sin\theta = 0$$

$$N = 0.3\,(490.5 + 200 \sin\theta)$$

$$(F_r)_{max} = \mu_s N$$

$$\therefore \quad (F_r)_{max} = 0.3\,(490.5 + 200 \sin\theta)$$

$$\sum F_x = 200 \cos\theta \,(\rightarrow)$$

a) For $\theta = 10°$,

$$(F_r)_{max} = 0.3\,(490.5 + 200 \sin 10°) = 157.57 \text{ N}$$

$$\sum F_x = 200 \cos 10° = 196.96 \text{ N} \,(\rightarrow)$$

$$\sum F_x > (F_r)_{max}$$

$\therefore$ Object moves towards right and

$$F_r = \mu_k N = 0.2 \times (490.5 + 200 \sin 10°)$$

$$\therefore \quad \boxed{F_r = 105.05 \text{ N}}$$

b) For $\theta = 40°$

$$(F_r)_{max} = 0.3\,(490.5 + 200 \sin 40°) = 185.72 \text{ N}$$

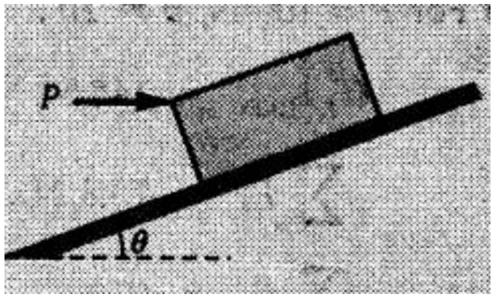
$$\sum F_x = 200 \sin 40 = 153.21 \text{ N} \,(\rightarrow)$$

$$\sum F_x < (F_r)_{max} \therefore \text{ Object doesn't move}$$

$$F_r = \sum F_x$$

$$\boxed{F_r = 153.21 \text{ N} \leftarrow}$$

3.Determine whether the 50 kg block shown in Fig. is in equilibrium and find the magnitude and direction of the frictional force when a) P = 200 N, θ =20° b) P = 200 N, θ = 45° c) P = 100 N, θ = 20°. μ_s = 0.3 and μ_k = 0.2.



Solution:

The F.B.D. of block is shown in Fig. (a) without frictional force

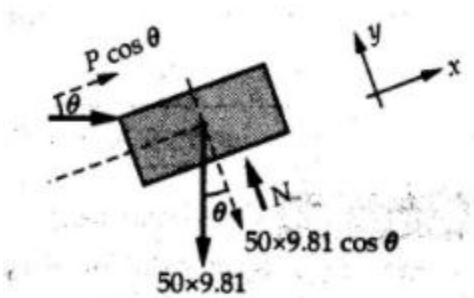


Fig (a)

a) For $P = 200$ N and $\theta = 20°$,

$(F_r)_{max} = 158.8$ N and $\sum F_x = 20.18$ N

As $\sum F_x$ is positive, the net force is directed upward along the inclined plane.

$\sum F_x < (F_r)_{max}$

Object does not move.

$F_r = \sum F_x$ in opposite direction.

$F_r = 20.18$ N, $\theta = 20°$ ↙

b) For $P = 200$ N, $\theta = 45°$

$$(F_r)_{max} = 146.48 \text{ N}, \quad \sum F_x = -205.41 \text{ N}$$

$$\therefore \quad \sum F_x = 205.41 \text{ N}, \; 45°$$

$$\sum F_x > (F_r)_{max}$$

$\therefore$ Object moves down the incline

$$F_r = \mu_k N = 0.2\,(200 \sin 45 + 490.5 \cos 45)$$

$$\therefore \quad \boxed{F_r = 97.65 \text{ N}, \; 45°}$$

c) For $P = 100$ N, $\theta = 20°$,

$$(F_r)_{max} = 148.54 \text{ N}, \quad \sum F_x = -73.8$$

$$\therefore \quad \sum F_x = 73.8 \text{ N}, \; 20°$$

$$\sum F_x < (F_r)_{max}$$

$\therefore$ Object does not move

$$F_r = \sum F_x \text{ in opposite direction}$$

$$\therefore \quad \boxed{F_r = 73.8 \text{ N}, \; 20°}$$