## 2.4 REFLECTION AND TRANSMISSION OF EM WAVES VACUUM - NON - CONDUCTING MEDIUM INTERFACE FOR NORMAL INCIDENCE

Let us consider a monochromatic (single frequency) uniform plane wave that travels through one medium (vacuum) and enters another medium (non-conducting) of infinite extent.

The uniform plane $EM$ wave propagating along $x$-direction in a vacuum medium $(\mu_o, \varepsilon_o)$ incident normally on the surface of a flat non-conducting medium permittivity, $\mu \neq \mu_o$ and permittivity, $\neq \varepsilon_0$ ).

Here the incoming EM wave is called the incident wave, the interface is an infinite plane at $x = 0$, the region to the left of the interface is medium $1(x \leq 0)$ and the region to the right of the interface is medium $2(x \geq 0)$.

At the interface, a part of the incident EM wave will penetrate the boundary (interface) and continue its propagation

Here $\langle \vec{s} \rangle$ is the propagation vector (or) poynting vector

Another remainder of the wave is reflected at the interface and then propagates in the negative $x$ direction. This wave is called the reflected wave

Thus both the incident and transmitted waves propagate in $+x$ direction. The reflected wave will propagate in $-x$ direction. So the incident and reflected waves are in medium 1 and the transmitted wave is in medium 2 .

Now by considering the electric field $\vec{E}$ of the incident wave which is polarized in $y$-direction (plane polarized) and has an amplitude $E_0$ at the interface as shown in fig. 2.21.

If $(k_1 = (1/v_1)$ is the propagation constant of this wave (with angular frequency $\omega$ and velocity equal to $v_1$ ) in medium-1, then the electric and magnetic field waves are represented as

$$\vec{E}_i(x,t) = E_0 \cos(\omega t - k_1 x)$$

and

$$\vec{B}_i(x,t) = \frac{E_o}{V_1}\cos(\omega t - k_1 x), \ldots \text{ (2)}$$

$$\left(\because B_o = \frac{E_o}{v_1}\right)$$

Then, the reflected waves are represented as,

$$\vec{E}_R(x,t) = E_1 \cos(\omega t + k_1 x),$$

and

$$\overrightarrow{B_R}(x,t) = \frac{E_1}{v_1}\cos(\omega t + k_1 x)$$

$$\vec{E}_T(x,t) = E_2 \cos(\omega t - k_2 x)$$

We know that $E_o = cB_o$

$$\vec{B}_T(x,t) = \frac{E_2}{v_2}\cos(\omega t - k_2 x)$$

Here in eqns (3) and (4), the sign is reversed used in the wave number $k$ to denote that this wave is propagating from the interface (boundary) along negative $x$ direction (backward travelling wave). Also the wave numbers $k_1$ and $k_2$ are related to

$$k_1 = \frac{\omega}{v_1}$$

and

$$k_2 = \frac{\omega}{v_2}$$

where $v_1$ and $v_2$ are the velocities of $EM$ waves in medium-1 and medium-2 respectively.

The total instantaneous electric field $\vec{E}_y$ for any value of $x$ with medium 1 is equal to the sum of the incident and reflected waves, so

$$\vec{E}_y(x,t) = E_o\cos(\omega t - k_1 x) + E_1\cos(\omega t + k_1 x)$$

(or)

$$\vec{E}_y(x,t) = \vec{E}_i(x,t) + \vec{E}_R(x,t)$$

The total instantaneous electric field $\vec{E}_y$ for any value of $x$ in the medium-2 is

$$\vec{E}_y(x,t) = E_2\cos(\omega t - k_2 x)$$

At the interface $x = 0$, the boundary conditions require that the tangential components of $\vec{E}$ and $\vec{B}$ fields must be continuous.

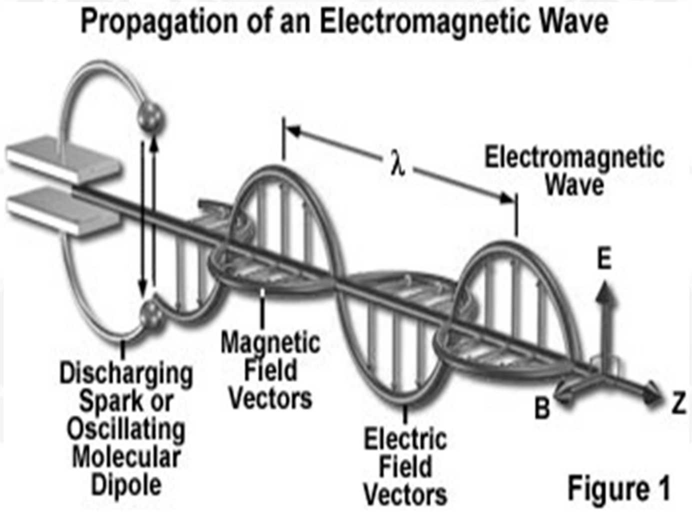


Fig. 2.22 (a) Electric field wave patterns

Fig. 2.22 (b) Magnetic field wave patterns

Since the waves are transverse, $\vec{E}$ and $\vec{B}$ fields are entirely tangential to the interface.

Hence at $x = 0$, eqn. (9) and (10) are equal, so

$E_0 \cos(\omega t - k_1 x) + E_1 \cos(\omega t - k_1 x) = E_2 \cos(\omega t - k_2 x)$

as $x = 0$, then

$$E_0 \cos(\omega t) + E_1 \cos(\omega t) = E_2 \cos(\omega t)$$

(or)

$$E_o + E_1 = E_2$$

Also at boundary $x = 0$, as tangential components are continuous therefore

$$\frac{dE_i}{dx} + \frac{dE_R}{dx} = \frac{dE_T}{dx}$$

which yields

$$-E_o k_1 \sin(\omega t) - E_1 k_1 \sin(\omega t) = E_2 k_2 \sin(\omega t)$$

(or)

$$E_o k_1 - E_1 k_1 = E_2 k_2$$

(or)

$$k_1 (E_o - E_1) = E_2 k_2,$$

(or)

$$E_o - E_1 = E_2 \cdot \left(\frac{k_2}{k_1}\right)$$

As $k_1 = \frac{\omega}{v_1}$ and $k_2 = \frac{\omega}{v_2}$, then eqn. (16) becomes

$$E_o - E_1 = E_2 \cdot \left(\frac{v_1}{v_2}\right)$$

Adding eqns (13) and (17) gives

$$\begin{aligned} 2E_o &= E_2 + E_2\left(\frac{v_1}{v_2}\right) \\ &= E_2\left(1 + \frac{v_1}{v_2}\right) \\ E_o &= \left(\frac{E_2}{2}\right)\left(1 + \frac{v_1}{v_2}\right) \end{aligned}$$

When medium-1 is vacuum $v_1 = c$, and $v_2 = v$

$$\therefore E_o = \left(\frac{E_2}{2}\right)\left(1 + \frac{c}{v}\right)$$

Subtracting eqn. (17) from eqn. (13) gives

$$E_1 = \left(\frac{E_2}{2}\right)\left(1 - \frac{v_1}{v_2}\right)$$

$$E_1 = \left(\frac{E_2}{2}\right)\left(1 - \frac{c}{v}\right)$$