4.10 Composite Stars

Composite stars are stellar remnants or systems formed after advanced stages of stellar evolution. These include **white dwarfs**, **neutron stars**, **black holes**, **supernovae**, **binary stars**, and **star clusters**.

2.1 White Dwarfs

- **Definition**: White dwarfs are remnants of low- to medium-mass stars that have exhausted their nuclear fuel and shed their outer layers, leaving behind a dense core.
- **Characteristics**:
    - Supported by **electron degeneracy pressure**, a quantum mechanical effect preventing electrons from being compressed beyond a certain limit.
    - Mass: Up to $1.4M_{\odot}$ (Chandrasekhar Limit).
    - Radius: ~10,000 km (comparable to Earth).
    - Density: A teaspoon of white dwarf material weighs ~5 tons.
    - Temperature: Can reach ~100,000 K initially but cools over billions of years.
    - Evolution: Over time, white dwarfs cool and fade into **black dwarfs**.

2.2 Neutron Stars

- **Definition**: Neutron stars are formed when the core of a massive star collapses under gravity during a supernova explosion.
- **Characteristics**:
    - Composed almost entirely of neutrons.
    - Supported by **neutron degeneracy pressure**.
    - Mass: $1.4–3M_{\odot}$.
    - Radius: ~10-15 km.
    - Density: Extremely dense; a sugar-cube-sized piece would weigh a billion tons.
    - Rotation: Many neutron stars rotate rapidly, emitting radiation as **pulsars**.
    - **Magnetars**: Neutron stars with extremely strong magnetic fields, up to 10^{15} Gauss.

2.3 Black Holes

- **Definition**: Black holes form when the core of a very massive star collapses to a point where gravity becomes so intense that not even light can escape.

- **Key Properties**:
  - **Event Horizon**: The boundary beyond which nothing can escape.
  - **Singularity**: A point of infinite density at the center.
  - Mass Types:
    - **Stellar-mass Black Holes** ($3-10M_{\odot}$).
    - **Intermediate-mass Black Holes** ($100-1000M_{\odot}$).
    - **Supermassive Black Holes** ($10^6-10^9M_{\odot}$), typically found at the centers of galaxies.
  - Einstein's theory of General Relativity governs their physics.

2.4 Supernovae

- **Definition**: A supernova is a powerful stellar explosion marking the end of a massive star's life.
- **Types**:
  - **Type I Supernovae**:
    - Occurs in binary systems where a white dwarf accretes enough mass to trigger a runaway nuclear reaction.
    - Example: Used as "standard candles" for distance measurement.
  - **Type II Supernovae**:
    - Results from core collapse in a massive star.
    - Leaves behind a neutron star or black hole.
- **Significance**:
  - Supernovae distribute heavy elements like iron, gold, and uranium into space, enriching the interstellar medium.

2.5 Binary Stars

- **Definition**: Binary stars consist of two stars orbiting a common center of mass.
- **Types**:
  - **Visual Binaries**: Resolvable with a telescope.
  - **Spectroscopic Binaries**: Detected via Doppler shifts in spectral lines.
  - **Eclipsing Binaries**: Periodic dimming as one star eclipses the other.
- **Importance**:
  - Binary systems help astronomers measure stellar masses and study interactions like mass transfer.

2.6 Star Clusters

- **Definition**: Groups of stars formed together from the same molecular cloud.
- **Types**:

- **Open Clusters**:
  - Found in the galactic disk; contain young, hot stars.
  - Example: The Pleiades.
- **Globular Clusters**:
  - Spherical clusters in the galactic halo; contain old stars.
  - Example: Omega Centauri.

Astrophysical Importance

Variable and composite stars serve as cosmic laboratories for studying:

1. **Distance Measurement**: Cepheids and Type Ia supernovae provide critical insights into the scale of the universe.
2. **Element Synthesis**: Supernovae and neutron star mergers produce heavy elements.
3. **Relativity and Quantum Mechanics**: Black holes and neutron stars test the limits of modern physics.

4.11 Chandrasekar Limit

The **Chandrasekhar Limit** is a critical mass threshold for white dwarf stars, beyond which the degeneracy pressure of electrons is no longer sufficient to counteract the gravitational collapse. This limit is approximately **1.44 solar masses ($M_\odot$)**, making it a pivotal concept in astrophysics, dictating the fates of stars and stellar remnants. Understanding this limit requires diving deeply into stellar physics, quantum mechanics, and general relativity.

What is the Chandrasekhar Limit?

The Chandrasekhar Limit defines the maximum mass of a stable **white dwarf**, a stellar remnant left after the death of low- to intermediate-mass stars. White dwarfs are supported by **electron degeneracy pressure**, a quantum mechanical force arising from the **Pauli Exclusion Principle**. This principle states that no two fermions (e.g., electrons) can occupy the same quantum state simultaneously.

When the mass of a white dwarf approaches this limit:

- The inward **gravitational force** exceeds the outward **electron degeneracy pressure**.
- This causes catastrophic collapse, leading to the formation of a **neutron star** or **black hole**, depending on the core mass.

Historical Context

- Proposed by **Subrahmanyan Chandrasekhar** in 1930, at the age of 19, during his journey to study at the University of Cambridge.
- His calculations were met with initial skepticism by renowned astrophysicists, including **Sir Arthur Eddington**, who opposed the idea of such limits to stellar stability.
- Over time, the concept gained acceptance through theoretical validations and observational evidence.

Chandrasekhar's work on this limit earned him the **Nobel Prize in Physics in 1983**, sharing the honor with William Fowler for their contributions to stellar evolution.

Theoretical Foundations

The Chandrasekhar Limit is rooted in the interplay between **quantum mechanics**, **hydrodynamics**, and **gravitational physics**.

1. Electron Degeneracy Pressure

White dwarfs are made of **electron-degenerate matter**, where the pressure arises not from thermal motion but from quantum mechanical constraints. This pressure is given by:

$$P_e \propto \rho^{5/3} \text{(Non-relativistic regime)}$$

Where:

- P_e is the electron degeneracy pressure,
- ρ is the mass density.

However, as the density increases, the electrons become relativistic (approaching the speed of light), and the pressure follows a different relationship:

$$P_e \propto \rho^{4/3} \text{(Relativistic regime)}$$

This weaker scaling in the relativistic regime means that electron degeneracy pressure can no longer balance gravity at extremely high densities.

2. Hydrostatic Equilibrium

A stable white dwarf maintains a balance between:

- **Inward Gravitational Pressure** (P_G):

$$P_G = \frac{GM(r)\rho}{r^2}$$

- Where, G is the gravitational constant, M(r) is the mass enclosed within radius r, and ρ is the density.
- **Outward Degeneracy Pressure** (P_e): Derived from quantum mechanics, as explained above.

This equilibrium can only persist below the Chandrasekhar Limit.

3. Relativistic Effects

As the star's mass increases:

- The density grows, forcing electrons to move relativistically.
- Relativistic electrons contribute less to degeneracy pressure, reducing their ability to counteract gravity.
- When the core mass exceeds **1.44 $M_\odot$**, collapse becomes inevitable.

Mathematical Derivation

Chandrasekhar derived the limit by solving the following equations under the conditions of hydrostatic equilibrium and relativistic degeneracy:

1. **Mass Continuity**:

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho$$

2. **Hydrostatic Equilibrium:**

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}$$

3. **Equation of State**:

$$P=K\rho^{\gamma}$$

   - For non-relativistic electrons, $\gamma=5/3$,
   - For relativistic electrons, $\gamma=4/3$.

Solving these equations yields the critical mass:

$$M_{\text{Ch}} = \frac{5.83}{\mu_e^2} M_\odot$$

Where, μ_e is the mean molecular weight per electron. For typical white dwarfs ($\mu_e \approx 2$), $M_{Ch} \approx 1.44 M_\odot$.

Astrophysical Implications

1. White Dwarfs

- White dwarfs below the Chandrasekhar Limit are stable.
- If a white dwarf in a **binary system** accretes matter and exceeds this limit, it collapses, triggering a **Type Ia Supernova**. These supernovae are "standard candles" used for measuring cosmic distances.

2. Neutron Stars

- If the collapsing core mass exceeds the Chandrasekhar Limit but is below the **Tolman-Oppenheimer-Volkoff (TOV) Limit** ($\sim 2-3 M_\odot$), the core forms a neutron star. Here, stability is provided by **neutron degeneracy pressure**.

3. Black Holes

- For core masses above the TOV Limit, even neutron degeneracy pressure fails, resulting in a **black hole**.

Observational Evidence

1. **Type Ia Supernovae**:
   - Observations confirm that these explosions occur when white dwarfs approach $1.44 M_\odot$, validating the Chandrasekhar Limit.
2. **Mass Measurements**:
   - White dwarfs observed in binary systems have masses consistent with the limit.
3. **X-ray and Gravitational Wave Observations**:
   - Neutron stars and black holes observed in systems further validate the collapse scenarios predicted by the Chandrasekhar Limit.

Relevance to Modern Physics

The Chandrasekhar Limit bridges multiple disciplines:

1. **Quantum Mechanics**: The role of electron degeneracy pressure.
2. **Relativity**: Effects of relativistic electrons at high densities.
3. **Astrophysics**: Stellar evolution, supernovae, neutron stars, and black holes.

Extensions and Challenges

1. Exotic States of Matter

- Beyond the Chandrasekhar Limit, alternative forms of matter, such as **quark matter** or **strange matter**, may exist.

2. Rotation Effects

- Rapid rotation can temporarily increase the mass limit for white dwarfs, delaying collapse.

3. Magnetic Fields

- Strong magnetic fields in some white dwarfs (e.g., magnetic white dwarfs) may alter their structural dynamics.