4.5. Maxwell's Equation

The First Maxwell's equation (Gauss's law for electricity)

Gauss's law states that flux passing through any closed surface is equal to $1/\epsilon_0$ times the total charge enclosed by that surface.

The integral form of Maxwell's 1st equation

$$\emptyset_E = \frac{q}{\epsilon_0} \text{ ------------ (1)}$$

$$Also \quad \emptyset_E = \int \vec{E}.d\vec{A} \text{ ------ (2)}$$

Comparing equation (1) and (2) we have

$$\int \vec{E}.d\vec{A} = \frac{q}{\epsilon_0} \text{ --------- (3)}$$

It is the integral form of Maxwell's 1st equation.

Maxwell's first equation in differential form

The value of total charge in terms of volume charge density is $q=\int \rho dv$ So equation (3) becomes

$$\int \vec{E}.d\vec{A} = \frac{1}{\epsilon_0} \int \rho dv$$

Applying divergence theorem on left hand side of above equation we have

$$\int (\vec{\nabla}.\vec{E}) d.V = \frac{1}{\epsilon_0} \int \rho dv$$

$$\int (\vec{\nabla}.\vec{E}) d.V - \frac{1}{\epsilon_0} \int \rho dv = 0$$

$$\int [(\vec{\nabla}.\vec{E}) - \frac{\rho}{\epsilon_0}] dv = 0$$

$$(\vec{\nabla}.\vec{E}) - \frac{\rho}{\epsilon_0} = 0$$

$$(\vec{\nabla}.\vec{E}) = \frac{\rho}{\epsilon_0}$$

It is called the differential form of Maxwell's 1st equation.

The Second Maxwell's equation (Gauss's law for magnetism)

Gauss's law for magnetism states that the net flux of the magnetic field through a closed surface is zero because monopoles of a magnet do not exist.

$\int \vec{B}.\overrightarrow{dA}=0$ .... (4)

It is the integral form of Maxwell's second equation.

Applying divergence theorem

$\int (\vec{\nabla}.\vec{B}) dV=0$

This implies that:

$\vec{\nabla}.\vec{B} =0$

It is called differential form of Maxwell's second equation.

The Third Maxwell's equation (Faraday's law of electromagnetic induction)

According to Faraday's law of electromagnetic induction

$\varepsilon = -N\frac{d\emptyset_m}{dt}$ .... (5)

Since emf is related to electric field by the relation

$\varepsilon = \int \vec{E}.\overrightarrow{dA}$

Also $\emptyset_m = \int \vec{B}.\overrightarrow{dA}$

Put these values in equation (5) we have

$\int \vec{E}.\overrightarrow{dA} = -N \int \vec{E}.\overrightarrow{dA} \int \vec{B}.\overrightarrow{dA}$

For N=1 ,we have

$\int \vec{E}.\overrightarrow{dA} = -\frac{d}{dt} \int \vec{B}.\overrightarrow{dA}$ .... (6)

It is the integral form of Maxwell's 3rd equation.

Applying Stokes Theorem on L.H.S of equation (6) we have

$\int(\vec{\nabla}\times\vec{E})d\vec{A} = -\frac{d}{dt}\int\vec{B}.\overrightarrow{dA}$

$\int(\vec{\nabla}\times\vec{E})d\vec{A} + \frac{d}{dt}\int\vec{B}.\overrightarrow{dA}=0$

$(\vec{\nabla}\times\vec{E})+\frac{d\vec{B}}{dt}=0$

$(\vec{\nabla}\times\vec{E})= -\frac{d\vec{B}}{dt}$

It is the differential form of Maxwell's third equation.

The Fourth Maxwell's equation (Ampere's law)

The magnitude of the magnetic field at any point is directly proportional to the strength of the current and inversely proportional to the distance of the point from the straight conductors is called Ampere's law.

$\int\vec{B}.\overrightarrow{ds}=\mu_0 i$ .... (7)

It is the integral form of Maxwell's 4 th equation.

The value of current density

$i = \int\vec{j}.\overrightarrow{dA}$

Now the equation (7) Become

$\int\vec{B}.\overrightarrow{ds}= \mu_0\int\vec{j}.\overrightarrow{dA}$

Applying Stoke's theorem on L.H.S of above equation, we have

$\int(\vec{\nabla}\times\vec{B})d\vec{A} = \mu_0\int\vec{j}.\overrightarrow{dA}$

$\int[(\vec{\nabla}\times\vec{B})d\vec{A} - \mu_0 j]. d\mathbf{A} =0$

$(\vec{\nabla}\times\vec{B}) = \mu_0\boldsymbol{j}$

Third Maxwell's equation says that a changing magnetic field produces an electric field. But there is no clue in the fourth Maxwell's equation whether a changing electric field produces a magnetic field? To overcome this deficiency, Maxwell's argued that if a changing magnetic flux can produce an electric field then by symmetry there must exist a relation in which a changing electric field must produce a changing magnetic flux. For more related informative topics Visit our Page: Electricity and Magnetism