

2.6 STEADY STATE ERROR

The deviation of the output of control system from desired response during steady state is known as steady state error. It is represented as e_{ss} . We can find steady state error using the final value theorem as follows.

$$e_{ss} = \lim_{t \rightarrow \infty} e(t)$$

$$e_{ss} = \lim_{s \rightarrow 0} sE(s)$$

where, $E(s)$ is the Laplace transform of the error signal, $e(t)$

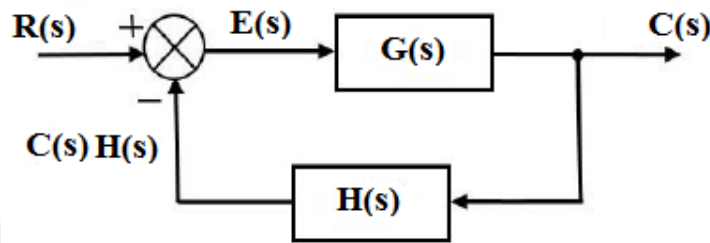


Figure 2.6.1 Closed loop control system

[Source: "Control Systems Engineering" by I J Nagrath, M Gopal, Page: 213]

$$C(s) = G(s)E(s)$$

$$E(s) = R(s) - C(s)H(s) = R(s) - G(s)E(s)H(s)$$

$$E(s)(1 + G(s)H(s)) = R(s)$$

$$E(s) = \frac{R(s)}{(1 + G(s)H(s))}$$

$$e_{ss} = \lim_{s \rightarrow 0} sE(s) = \lim_{s \rightarrow 0} \frac{sR(s)}{(1 + G(s)H(s))}$$

When a control system is excited with standard input signal, the steady state error may be zero, constant or infinity. Its value depends on the type number and input signal.

- Type-0 system will have a constant steady state error when the input is step signal
- Type-1 system will have a constant steady state error when the input is ramp signal
- Type-2 system will have a constant steady state error when the input is parabolic signal

$$\text{For unit step input, } e_{ss} = \frac{1}{1+K_p}$$

$$\text{For unit ramp input, } e_{ss} = \frac{1}{K_v}$$

$$\text{For unit parabolic input, } e_{ss} = \frac{1}{K_a}$$

Static error constants for various type number of systems

Error constants	Type number of system			
	0	1	2	3
K_p	Constant	∞	∞	∞
K_v	0	Constant	∞	∞
K_a	0	0	Constant	∞

Steady state error for various types of input

Input signal	Type number of system			
	0	1	2	3
K_p	$\frac{1}{1 + K_p}$	0	0	0
K_v	∞	$\frac{1}{K_v}$	0	0
K_a	∞	∞	$\frac{1}{K_a}$	0

