Statics of Rigid bodies Force couple system

Moment of a Force:-

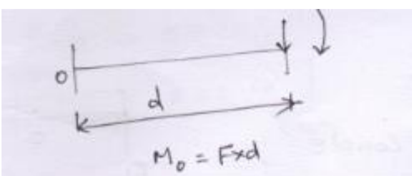
Moment of a Force about a point is defined as the product of the force and the perpendicular distance of the line of action of the force from the point

$M = F \times d$

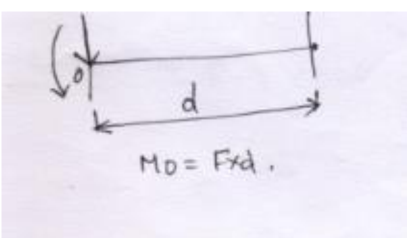
$F = Force$

$d = perpendicular\ distance$

The clockwise direction of moment is positive direction of moment



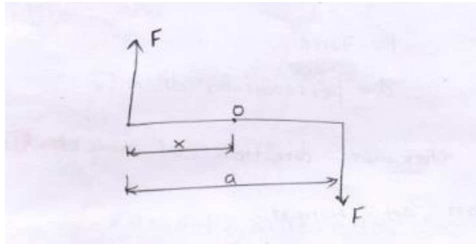
The Anticlockwise bending moment gives the negative direction of moment



Coupled force:

It is a turning effect produce in the body of object by applying two forces having same magnitude put in opposite Direction.

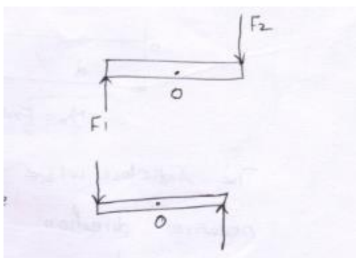
Two forces F and –F having the same magnitude, parallel lines of action and opposite sense are said to form a couple.



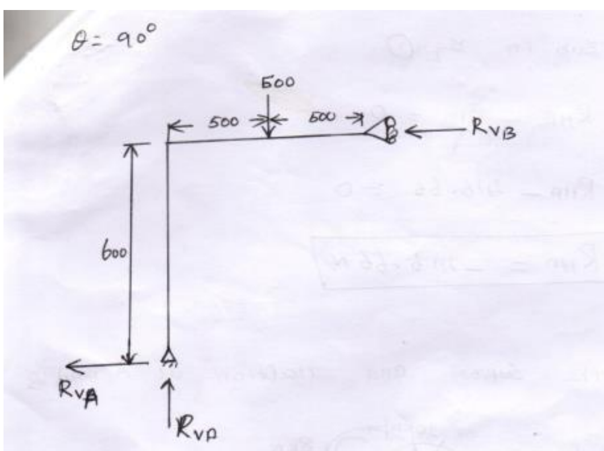
Types of couple:

1. clock wise couple

2. Anti- clockwise couple



$\theta = 90°$



ΣF_H =0

$-RVA - RVB = 0$_________(1)

$\sum FV = 0$

$RV_A - 500 = 0$

$RV_A = 500N$

$\sum M_A = 0$

$[500 \times 500] + [-RV_B \times 600] = 0$

$250 \times 103 = RV_B \times 600 = 0$

$-RV_B = -250 \times 103$

$$RV_B = \frac{-250 \times 10^3}{-600}$$

$RV_B = 416.66N$

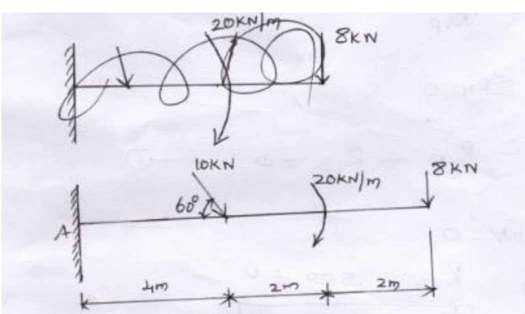
R_{VB} Sub in Eqn (1)

$-RH_A - RH_B = 0$

$RH_A = -416.66N$

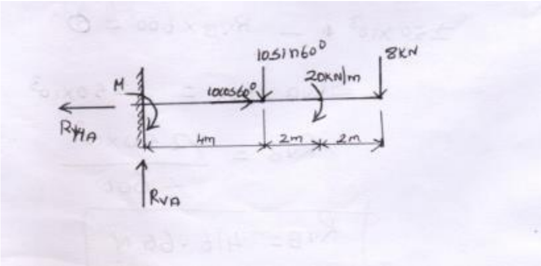
Problem:

Determine the support and reaction at A and B



Given

Free body diagram



$-1019.61\, RV_B = -250 \times 103$

$RV_B = \dfrac{-250 \times 10^3}{-1019.61}$

$RV_B = 245.19\, N$

Sub in Eqn------ (1)

$-RH_A - RV_B \cos 30 = 0$

$-RH_A = -RV_B \cos 30 = -245.19 \cos 30$

$RH_A = 212\, N$

RV_A *Sub in* (2)

$RV_A + \sin 30 = 500$

$RV_A = -RV_B \sin 30 + 500$

$RV_A = -245.19 \times \sin 30 + 500$

$RV_A = -122.5 + 500$

$RV_A = 377.5 N$

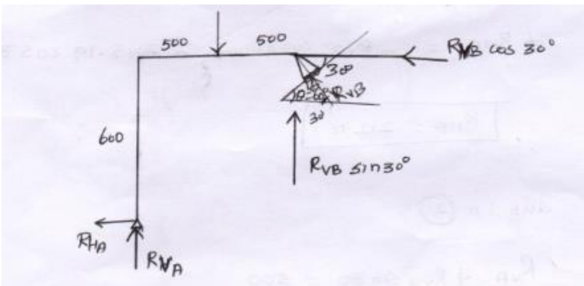
$RV_B = \dfrac{-250 \times 10^3}{-1000}$

$RV_B = 250\, N$

$RV_A + RV_B = 500 ---> RV_A = 500 - 250$

$RV_A = 250N$

ii) when θ=60°



$\sum F_H = 0$

$-RH_A - RV_B cos\,30 = 0$

$\sum F_v$ =0

$RV_A - 500 + Sin\,30 = 0$

$RV_A + RV_B\ Sin\,30° = 500$

$\sum M_A$ =0

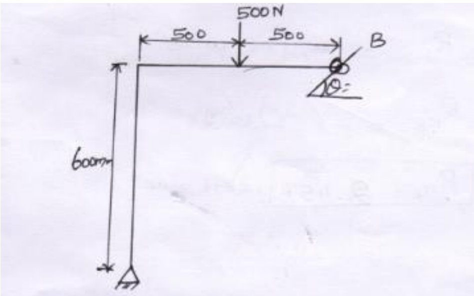
$\sum M_A$ =[500× 500] + RV_B cos 30 × 600] + [$-RV_B$ sin 30° × 1000]

$$\sum M_A = 250 \times 10^3 - RV_B\,519.61 - RV_B\,500 = 0$$

250×10^3 −01019.61 RV_B =0

Problem:

A Frame supported at A and B is subjected to force 500N as shown in fig compute the Reaction the support for the cases i) θ =90°θ =60°



Given $\theta = 0° \theta = 90° \theta = 60°$

To find

Reaction at the support

i) $\theta = 0^0$

$\sum F_H = 0$

$RH_A = 0$

$\sum F_V = 0$

$-500 + RV_B + RV_A = 0$

$RV_A + RV_B = 500$------> (1)

$\sum M_A = 0$

$[500 \times 500] + [RV_B \times 1000] = 100$

$250 \times 10^3 - 1000 RV_B = 0$

$-1000 RV_B = -250 \times 10^3$

To find reaction 'R'

$\sum F_H = 0$

$$-RH_R - F_{PQ}\cos 25^0 = 0$$

$$-RH_R = F_{PQ} cos25^\circ$$

$$RH_R = -F_{PQ} cos25^\circ$$

$$RH_R = - \times 3 \times cos25$$

$$RH_R = 2.45\, N$$

$$\sum F_V = 0$$

$$RV_R - 4 - FP_Q sin\, 23 = 0$$

$$RV_R - 4 - 3 \times 25^\circ = 0$$

$$RV_R = 4 + 3sin\, 25^\circ$$

$$RV_R = 5.26\, N$$

$$\sqrt{[RH_R]^2 + [RV_R]^2}$$

$$R = \sqrt{(2.45)^2 (5.26)^2}$$

$$R = 5.80N$$

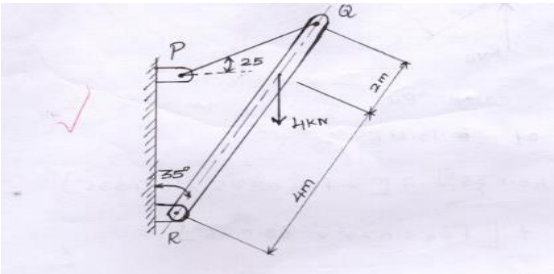
$$\theta = tan^{-1}\left(\frac{\sum R_H}{\sum R_V}\right)$$

$$\theta = tan^{-1}\frac{5.26}{2.43}$$

$$\theta = 65^0$$

Problem:

4000N load acts on the beam held by a cable PQ as shown in fig. The weight of the beam can be neglected. Draw the free body diagram of the beam and find tension in cable PQ. Also find the reaction force at R



To find:

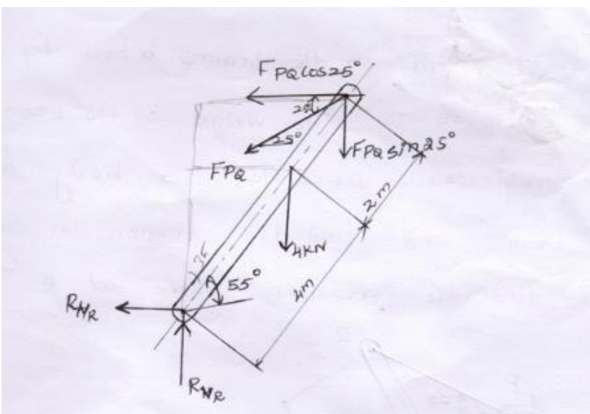
1. Free body diagram

2. Tension in cable PQ

3. Reaction on Force R

Soln:

1. Free body diagram:



2. Tension in cable 'PQ'

Moment at point 'R'

$$\sum M_R = [4 \times \sin 35^\circ] + [-F_{PQ} \cos 25^\circ \times 6 \cos 35^\circ] + [F_{PQ} \sin 25^\circ \times 6\ sin 35^\circ] = 0$$

$$\sum M_R = 9.177 - F_{PQ} \times 4.454 + 1.45 F_{PQ} = 0$$

$$-4.45F_{PQ} + 1.45F_{PQ} = -9.177$$

$$-3F_{PQ} = -9.177$$

$$F_{PQ} = \frac{-9.177}{-3}$$

$$F_{PQ} = 3N$$

Procedure for finding out the resultant of non current coplander force system:

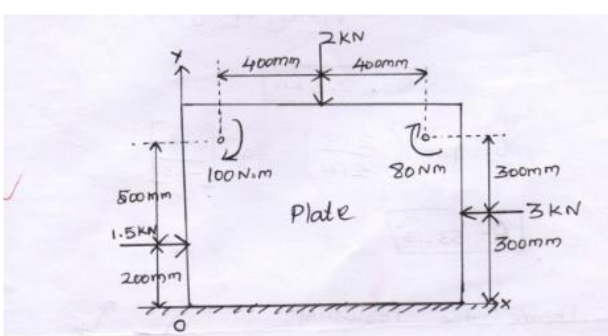
1. Resolve the given forces, if they are inclined to reference x and y Axis.
2. Find the sum of horizontal component of forces $\sum FH$
3. Find the sum of vertical component of forces $\sum FV$
4. 4.Calculate the resultant force R =$\sqrt{(\sum(F_H)^2 + (\sum(F_V)}$2
5. Angle of inclination of resultant θ =tan^{-1}[$\frac{\sum FV}{\sum FH}$]
6. If the force moment system is converted into a single force, coordinate position is given by

$$\sum M_o = R \times x$$

$$\sum M_o = \sum F_v \times x$$

$$\sum M_o = \sum F_H \times y$$

A plate os acted upon by three force and two couple as shown in fig. determine the resultant of these force couple system and find co-ordinate x of the point on the x axis through which the resultant is passed



Given

Three force $1.5KN, 2KN, 3KN$

Two couple $100N.m$ $80\ N.m$

To find

Resultant force, location

Soln:

Resultant force R = $\sqrt{(\sum(F_H)^2 + (\sum(F_V)2}$

Sum of horizontal

$$\sum F_H = 0$$

$\sum F_H = 1.5 - 3$

$\sum F_H = -1.5KN1$

Sum of vertical force $\sum Fv$ =0

$\sum F_V = -2\ KN$

Resultant $R = \sqrt{(\sum(F_H)^2 + (\sum(F_V)}$

$$R = \sqrt{[-1.5]^2 [-2]^2}$$

$R = 2.5KN$

$\theta = tan^{-1}\left[\frac{\sum F_V}{\sum F_V}\right] = tan^{-1}\left[\frac{-2}{-1.5}\right]$

$\theta = 53.13°$

To locate the resultant

By varigon's Thorem $\downarrow + \uparrow -$

$\sum M_o = R \times x \; and \sum M_o = \sum F_y \times x$

$\sum M_O = [3 \times 0.3] + [-2 \times 0.5] + [-1.5 \times 0.2] + [-0.1] + [-0.08] = 0$

$\sum M_O = -0.58 \; KN.M$

$\sum Mo = 0.58 \; KN.M$ [clock wise]

The co-ordinate x of the point through which the resulted passes is given by

$\sum$Mo= $\sum FY \times x$ $x = \frac{0.58}{2}$

$0.58 = 2 \times x$

$x = 0.29m$

$x = 290mm$

we want to find the intersection

$\sum M_o = \sum F_H \times y$

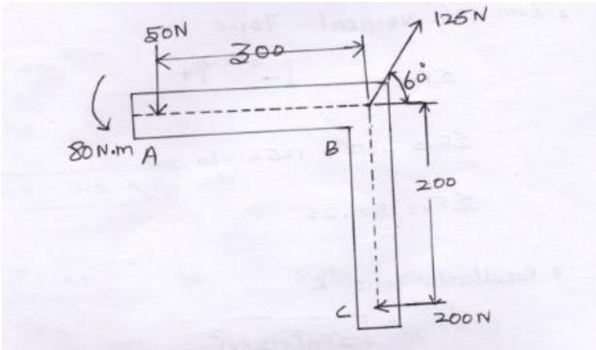
$0.581.5 \times y$

$y = 0.387 \; m$

The three forces and a couple shown below are applied to an angel bracket

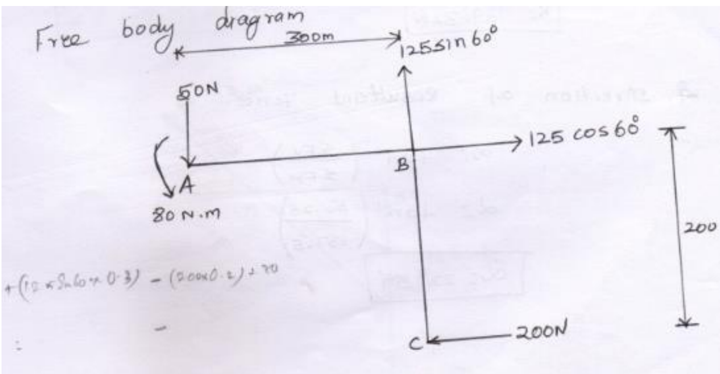
(i) Find he Resultant of this system of forces

(ii) Locate the points where the line o action of the resultant intersects line AB and the line BC



Soln

Free body diagram



1. Sum of Horizontal force

$$\sum F_H = 0 \qquad \overset{+}{\rightarrow}\underset{-}{\leftarrow}$$

$$\sum F_H = +125 \cos 60 - 200 = 0$$

$$\sum F_H = -137.5N$$

2. Sum of Vertical Force

$$\sum F_V = 0 \downarrow - \ \uparrow +$$

$$\sum F_V = -50 + 125 \sin 60 = 0$$

$$\sum F_V = 58.25$$

3. Resultant force' R'

$$R = \sqrt{(\sum(F_H)^2 + (\sum(F_V)^2}$$

$$R = \sqrt{[-137.5]^2 + [58.23]^2}$$

$$R = 149.32N$$

4. Direction of Resultant force α

$$\alpha = \tan^{-1}(\frac{\sum FV}{\sum FV})$$

$$\alpha = \tan^{-1}(\frac{58.25}{137.5})$$

$$\alpha = 22°57'$$

Location of Resultant Force:

By Varigon's Theorem

$$\sum M_A = \sum F_V \times x \; and \sum M_A = \sum F_H \times y$$

$$\sum M_A = (200 \times 0.2) + (-125 \sin 60 \times 0.3) - 80:0$$

$$\sum M_A = 40 - 32.47 - 80$$

$$\sum M_A = -7.5\, N.m$$

$$\sum M_A = \sum F_V \times x$$

$7.5 = 58.25 \times x$

$x = 7.5/58.25 = 0.12\ m$

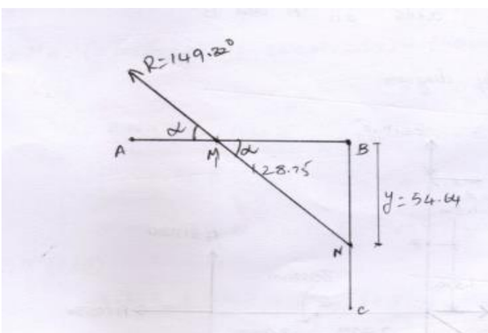
$x = 128.75mm$

$\sum M_A = \sum F_v \times y$

$7.5 = 137.25 \times y$

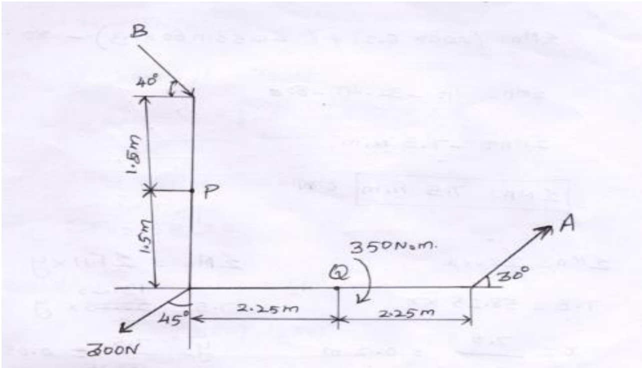
$y = 7.5/137.25 = 0.05m$

$y = 54.64\ mm$



Problem:

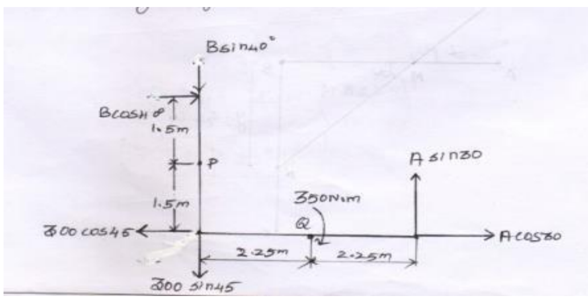
A system of forces acts as shown in fig.find the magnitude of A and B so that resultant of the force system passes through P and Q



To Find:

Forces acts on A and B

Soln: Free body diagram



The resultant forces passes through P and Q is moment About pis zero and also moment about Q=0

It only means that the algebraic sum of moment about P and Q is equal to zero

$\sum M_P = 0$ $\quad\quad\downarrow + \ \uparrow -$

$$\sum M_P = (+B\ \cos 40 \times 1.5) + (300 \cos 45 \times 1.5) + 350 + (-A \sin 30 \times 4.5) + (-A \cos 30 \times 1.5) = 0$$

$$\sum M_P = 1.149\,B + 318.19350 - 2.25A - 1.29$$

$$\Sigma M_P = 1.49B + 668.19 - 3.54A = 0$$

$-3.54A + 1.149B = -668.19$______>(1)

$\sum M_Q = 0 \quad \downarrow + \uparrow -$

$\sum M_Q = (B\cos 40 \times 3) + (-B\sin 40 \times 2.25) + (-300\sin 45 \times 2.25) + 350 + (-A\sin$
30×2.25)=0

$2.29B - 1.44B - 477 + 350 - 1.125A = 0$

$0.85B - 127 - 1.125A = 0$

$-1.25A + 0.85B = 127 - - - - - - - - - - -> (2)$

Solve 1&2

$-3.54A + 1.149B = -668.19$____(1)

$-1.125A + 0.85B = 127$______(2)

$(1) \times 1.25 \;\; => -3.982A + 1.292B = -751.7$

$(2) \times 3.54 \;\; => \;\; 3.982A + \; (-)\, 3.009B =< -> 449.58$

- -

$-171B = -1201.29$

$B = (-1201.29)/(-1.71)$

$B = 702.508N$

B Value substituting in Eqn (1)

$-3.54A + 1.149 \times 702.508 = -668.19$

$-3.54A + 807.182 = -668.19$

$-3.54A = -668.19 - 807.182$

$-\,3.54A = -1475.37$

$A = (-1475.37)/(-3.54)$

$A = 416.77\,N$

Result:-

$Force\ on\ A = 416.77N$

$Force\ on\ B\ = 702.508N$

Take moment about 'A'

$\sum M_A$=0

$\sum M_A = (500 \times 11) + (-200 \times 7) + (1200 \times 5) + (-300 \times 2)$

$\sum M_A = 5500 = 1400 + 6000 - 600$

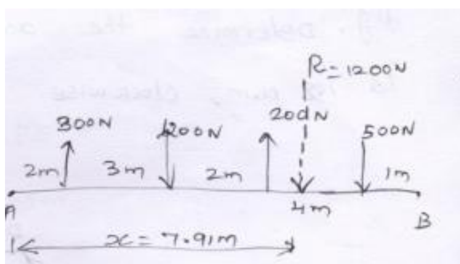
$\sum M_A = 9500\ N.m$

By varignon's theorem

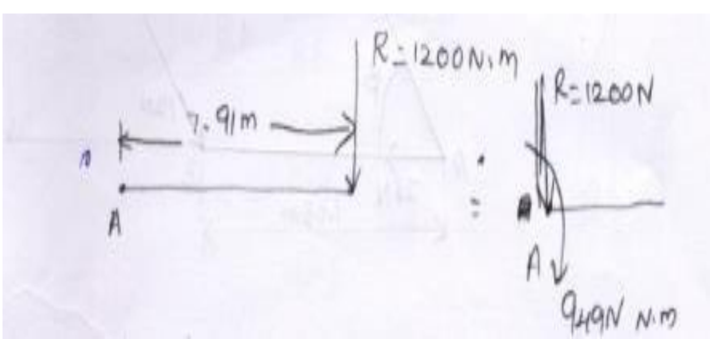
$\sum M_A = R \times x$

$9500 = 1200 \times x$

$x = 7.91m$



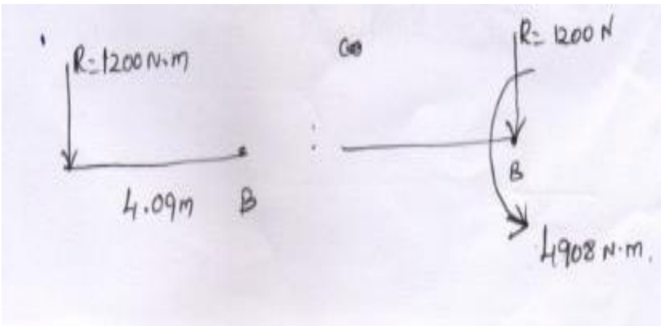
Force couple systemant 'A'



$$Couple\ at\ A = 1200 \times 7.91$$

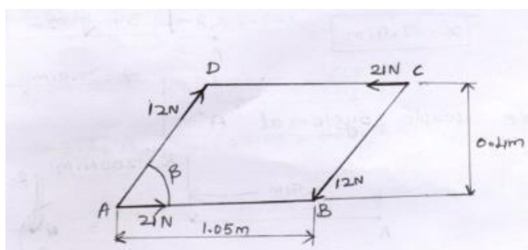
$$A = 9492\ N.m$$

Couple system at B



Problem:

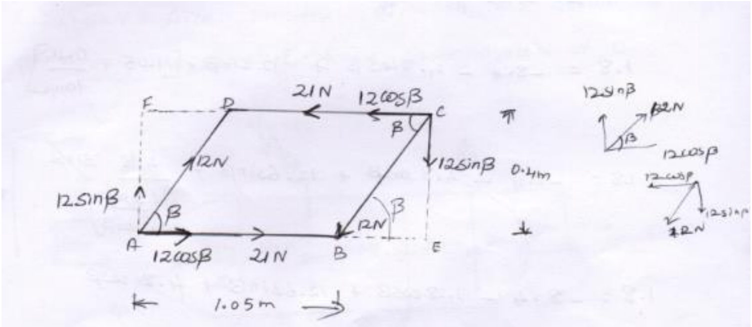
A plate ABCD in the shape of parallelogram is acted upon the two couples, as shown in the fig. Determine the angle B if the resultant couple is 1.8 N.m clockwise



Given:

Resultant couple =1.8N. m

Free body diagram



$Distance\ of\ AE = AB + BE$

$AB = 1.05\ m$

To find BE

$tan\beta = \frac{CE}{BE} = \frac{0.4}{BE}$

$BE = \ 0.4/tan\ \beta$

$AE = AB + BE$

$AE = 1.05 + \frac{0.4}{tan\,\beta}$

Given the resultant couple $\sum M_A = 1.8 N.M$

Take moment about A

$\sum M_A = [-21 \times 0.4] + [-12\ cos\beta \times 0.4] + [12 sin\beta \times AE]$

$\sum M_A = 1.8\ N.M$

$1.8 = -8.4\ - 4.8cos\beta + 12\ sin\beta \times [1.05 + \frac{0.4}{tan\beta}]$

$1.8 = -8.4\ - 4.8cos\beta + 12.6\ sin\beta + \frac{4.8}{\frac{sin\beta}{cos\beta}}\ sin\ \beta$ $\qquad tan\theta = \frac{sin\,\theta}{cos\,\theta}$

$$1.8 = -8.4 - 4.8\,cos\beta\ + 12.6\,sin\,\beta + 4.8\,cos\,\beta$$

$$1.8 + 8.4 = -4.8\,cos\beta + \ 12.6\,sin\beta\ + 4.8cos\,\beta$$

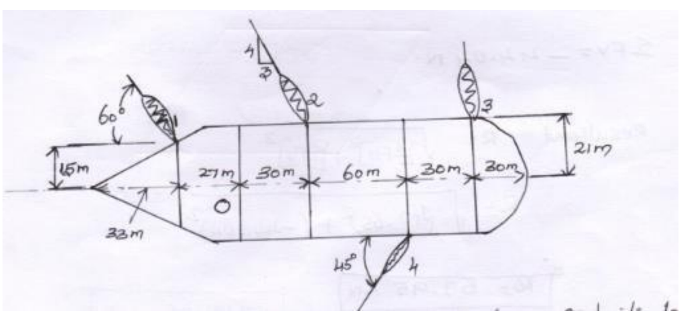
$$10.2\ = 12.6sin\beta$$

$$Sin\beta = \frac{10.2}{12.6}$$

$$B = sin - 1\left(\frac{10.2}{12.6}\right) \qquad B = 54^{\circ}$$

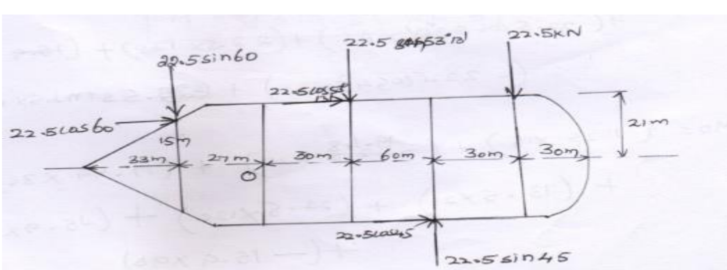
Problem

Four tugboats are used to bring an ocan large ship to us pier. Each tugboat exerts a 22.5KN force in direction as shown in fig (i) determine the equivalent force couple system at ‘o’



(ii) Determine a single equivalent force and its location along the longitudinal axis of the ship

Soln: Free body diagram



$$\sum F_H = 22.5\ cos\ 60 + \ 22.5\ cos + \ 53^{\circ} + 22$$

$\sum F_H$=$40.65KN$

$\sum F_v = -22.5\ sin\ 60 - 22.5 + \ sin\ 53° + 22.5\ sin\ 45 - 22.5$

$\sum F_v = -44.04N$

Resulant R= $\sqrt{(\sum(F_H)^2 + (\sum(F_V)}$2

$R = \sqrt{[40.6.5]^2 + [-44.04]^2}$

$R = 59.95KN$

Direction $\theta = tan^{-1}\left(\frac{\sum F_V}{\sum F_H}\right) = tan^{-1}\left[\frac{44.04}{40.65}\right] = 47°3'$

To find location:

$\sum M_o = R \times x$

$\sum M_o = (22.5cos\ 60\ \times 15) + (-22.5sin\ 60 \times 27) + (22.5sin\ 53° \times 30) +$

$(22.5\ cos\ 53°\ 31 \times 21) + (22.5 \times 120) + (-22.5\ cos\ 45 \times 21) + (-22.5\ \ 45 \times 90)$

$\sum M_o = (11.25 \times 15) + (-19.48 \times 27) + (17.99 \times 30) + (13.5 \times 21) +$

$(22.5 \times 120) + (-15.9 \times 21) + (-15.9 \times 90)$

$\sum M_o = 1319.5\ KN.m$

Location

$\sum M_o = 1319.5$

$\sum M_o = R \times x$

$x = 1319/59.95$

$x = 22.01m$

Magnitude of couple

$M = R \times x$

$= 59.95 \times 22.01m$

$M = 1319.55\ KN.m$