PELTON WHEEL

A Pelton wheel/turbine consists of a **rotor**, at the periphery of which are mounted equally spaced *double hemispherical or double ellipsoidal buckets*. Water is transferred from a high head source through penstock which is fitted with a **nozzle**, through which the water flows out at a *high speed jet*. **A needle spear** moving inside the nozzle *controls the water flow through the nozzle* and the same time, provides a smooth flow with negligible energy loss. All the available *potential energy is thus converted into kinetic energy* before the jet strikes the **buckets** of the **runner**. *The pressure all over the wheel is constant and equal to atmosphere, so that energy transfer occurs due to purely impulse action.*

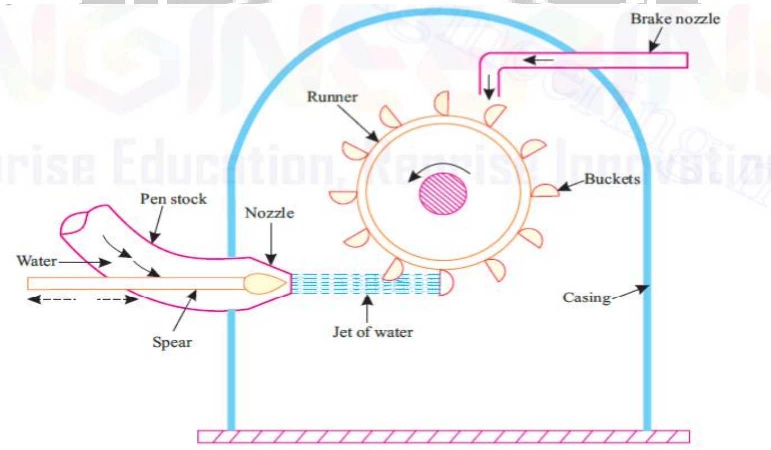


Fig. 2.2. Pelton wheel.

Work done and Efficiency of a Pelton Wheel

Fig. 2.5 shows the velocity triangles.

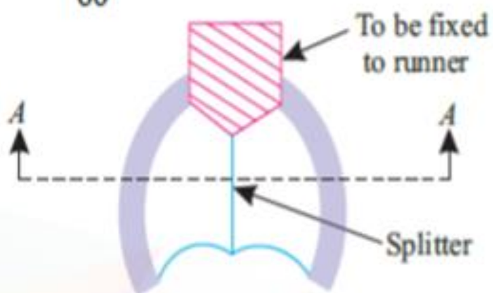
Let, N = Speed of wheel in r.p.m.,

D = Diameter of the wheel,

d = Diameter of the jet,

u = Peripheral (or circumferential) velocity of runner. It will be same at inlet and outlet of the runners at the mean pitch. (*i.e.* $u = u_1 = u_2$)

$$= \frac{\pi DN}{60},$$



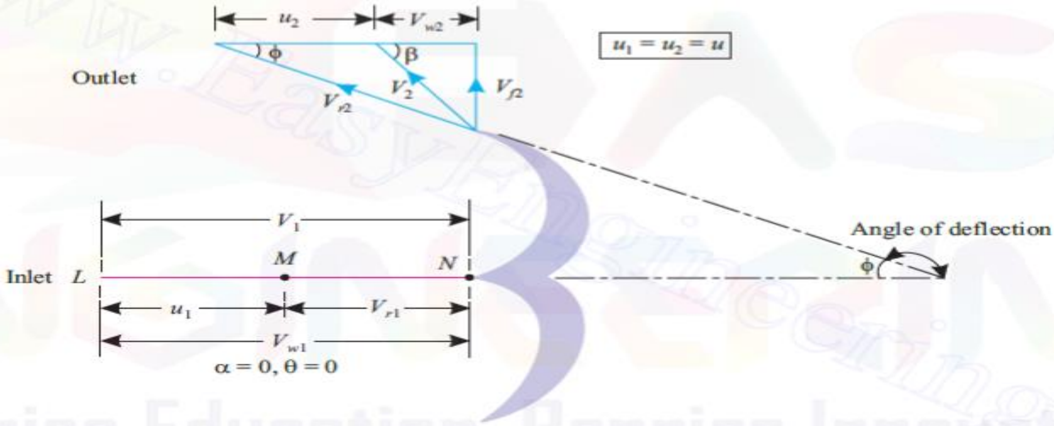


Fig. 2.5. Velocity triangles.

V_1 = Absolute velocity of water at inlet,

V_{r1} = Jet velocity relative to vane/bucket at inlet,

α = Angle between the direction of the jet and direction of motion of the vane/bucket (also called *guide angle*),

θ = Angle made by the relative velocity (V_{r1}) with the direction of motion at inlet (also called *vane angle at inlet*),

V_{w1} and V_{f1} = The components of the velocity of the jet V_1, in direction of motion and perpendicular to the direction of motion of the vane respectively;

V_{w1} is also known as *velocity of whirl* at inlet,

V_{f1} is also known as *velocity of flow* at inlet,

V_2 = Velocity of jet, leaving the vane or velocity of jet at outlet of the vane,

V_{r2} = Relative velocity of the jet with respect to the vane at outlet,

ϕ = Angle made by the relative velocity V_{r2} with the direction of motion of the vane at outlet and also called *vane angle at outlet*,

β = Angle made by the velocity V_2 with the direction of motion of the vane at outlet, and

V_{w2} and V_{f2} = Components of the velocity V_2, in the direction of motion of vane and perpendicular to the direction of motion of vane at outlet;

V_{w2} is also called the *velocity of whirl at outlet*, and

V_{f2} is also called the *velocity of flow at outlet*.

Inlet. The velocity triangle at *inlet* will be a *straight line* where

$$V_{r1} = V_1 - u_1 = V_1 - u, \; V_{w1} = V_1 \qquad (\because u_1 = u_2 = u)$$

$$\alpha = 0 \text{ and } \theta = 0$$

Outlet : From velocity triangle at outlet, we have

$$V_{r2} = KV_{r1},$$

[ρ and a are the mass density and area of jet $\left(a = \frac{\pi}{4} d^2\right)$ respecitvely.]

Now work done by the jet on runner per second

$$= F \times u = \rho a V_1 (V_{w1} + V_{w2}) \times u \qquad ...(2.2)$$

Work done per second per unit weight of water striking

$$= \frac{\rho a V_1 (V_{w1} + V_{w2}) \times u}{\text{Weight of water striking}} = \frac{\rho a V_1 (V_{w1} + V_{w2}) \times u}{\rho a V_1 \times g}$$

$$= \frac{1}{g} (V_{w1} + V_{w2}) u \qquad ...[2.2\ (a)]$$

The energy supplied to the jet at inlet is in the form of K.E. and is equal to $\frac{1}{2} m V_1^2$.

$\therefore$ Kinetic energy (K.E.) of jet per second $= \frac{1}{2} (\rho a V_1) \times V_1^2$

$\therefore$ Hydraulic efficiency, $\eta_h = \dfrac{\text{Work done per second}}{\text{K.E. of jet per second}} = \dfrac{\rho a V_1 (V_{w1} + V_{w2}) \times u}{\frac{1}{2} (\rho a V_1) \times V_1^2}$

or,

$$\eta_h = \frac{2 (V_{w1} + V_{w2}) \times u}{V_1^2} \qquad ...(2.3)$$

From inlet and outlet velocity triangles, we have:

$$V_{w1} = V_1,\ V_{r1} = V_1 - u_1 = V_1 - u$$

$$V_{w2} = V_{r2} \cos\phi - u_2 = V_{r2} \cos\phi - u = K V_{r1} \cos\phi - u = K (V_1 - u) \cos\phi - u$$

Substituting the values of V_{w1} and V_{w2} in eqn (2.3), we have:

$$\eta_h = \frac{2[V_1 + K(V - u)\cos\phi - u]u}{V_1^2} = \frac{2[(V_1 - u)(1 + K\cos\phi)]u}{V_1^2} \qquad ...(2.4)$$

The hydraulic efficiency will be *maximum* for given value of V_1 when,

$$\frac{d}{du}(\eta_h) = 0$$

i.e.,

$$\frac{d}{du}\left[\frac{2 (V_1 - u)(1 + \cos\phi) u}{V_1^2}\right] = 0$$

or,

$$\frac{2 (1 + K \cos\phi)}{V_1^2} \times \frac{d}{du}(V_1 u - u^2) = 0$$

Since,

$$\frac{2 (1 + K \cos\phi)}{V_1^2} \neq 0,\ \therefore\ \frac{d}{du}(V_1 u - u^2) = 0$$

or,

$$V_1 - 2u = 0, \text{ or, } u = \frac{V_1}{2} \qquad ...(2.5)$$

The above equation states that *hydraulic efficiency of a Pelton wheel is maximum when the velocity of the wheel is half the velocity of jet of water at inlet.* The maximum efficiency can be obtained by substituting the value of $u = \frac{V_1}{2}$ in eqn. (2.4).

$$(\eta_h)_{max} = \frac{2\left(V_1 - \frac{V_1}{2}\right)(1 + K\cos\phi)\frac{V_1}{2}}{V_1^2} = \frac{2 \times \frac{V_1}{2}(1 + K\cos\phi) \times \frac{V_1}{2}}{V_1^2}$$

or,

$$(\eta_h)_{max} = \frac{(1 + K\cos\phi)}{2} \qquad ...(2.6)$$

If friction factor, $K = 1$ (*i.e.,* assuming *no friction*), we have

$$(\eta_h)_{max} = \frac{1 + \cos\phi}{2} \qquad ...[2.6(a)]$$

18.6.2 Points to be Remembered for Pelton Wheel

(*i*) The velocity of the jet at inlet is given by $V_1 = C_v\sqrt{2gH}$

where C_v = Co-efficient of velocity = 0.98 or 0.99

H = Net head on turbine

(*ii*) The velocity of wheel (u) is given by $u = \phi\sqrt{2gH}$

where ϕ = Speed ratio. The value of speed ratio varies from 0.43 to 0.48.

(*iii*) The angle of deflection of the jet through buckets is taken at 165° if no angle of deflection is given.

(*iv*) The mean diameter or the pitch diameter D of the Pelton wheel is given by

$$u = \frac{\pi DN}{60} \text{ or } D = \frac{60u}{\pi N}.$$

(*v*) **Jet Ratio.** It is defined as the ratio of the pitch diameter (D) of the Pelton wheel to the diameter of the jet (d). It is denoted by 'm' and is given as

$$m = \frac{D}{d} \text{ (= 12 for most cases)} \quad \text{...(18.16)}$$

(*vi*) Number of buckets on a runner is given by

$$Z = 15 + \frac{D}{2d} = 15 + 0.5 \text{ m} \quad \text{...(18.17)}$$

where m = Jet ratio

(*vii*) **Number of Jets.** It is obtained by dividing the total rate of flow through the turbine by the rate of flow of water through a single jet.

Example 2.1. *A Pelton wheel is receiving water from a penstock with a gross head of 510 m. One-third of gross head is lost in friction in the penstock. The rate of flow through the nozzle fitted at the end of the penstock is 2·2 m³/s. The angle of deflection of the jet is 165°. Determine :*

(*i*) *The power given by water to the runner, and*

(*ii*) *Hydraulic efficiency of the Pelton wheel.*

Take C_v (co-efficient of velocity) = 1·0 and speed ratio = 0·45.

Solution. Gross head, $H_g = 510$ m

Head lost in friction, $h_f = \frac{H_g}{3} = \frac{510}{3} = 170$ m

$\therefore$ Net head, $H = H_g - h_f = 510 - 170 = 340$ m

Discharge, $Q = 2{\cdot}2 \text{ m}^3/\text{s}$

Angle of deflection = 165°

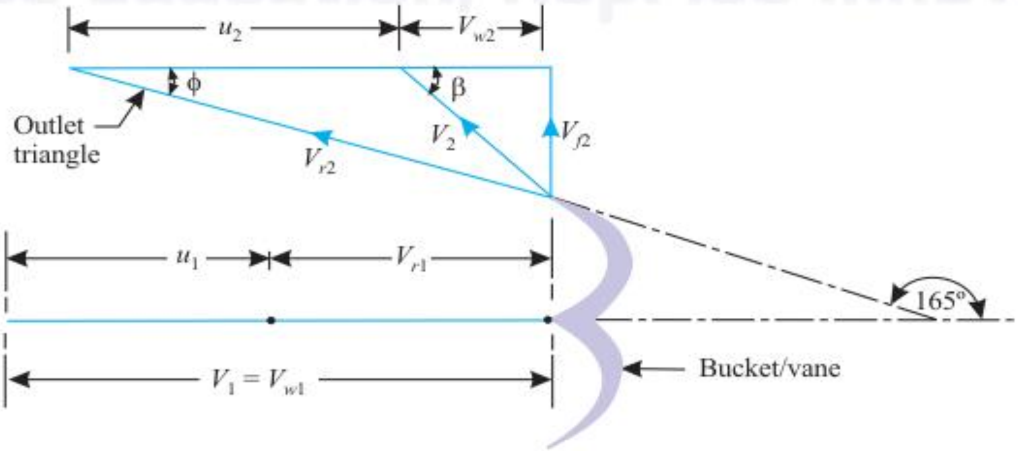


Fig. 2.7

$\therefore$ Angle, $\phi = 180° - 165° = 15°$

Co-efficient of velocity, $C_v = 1.0$

Speed ratio, $K_u = 0.45$

(*i*) The power given by water to the runner :

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 1.0\sqrt{2 \times 9.81 \times 340} = 81.67$ m/s

Velocity of wheel, $u = K_u\sqrt{2gH} = 0.45\sqrt{2 \times 9.81 \times 340} = 36.75$ m/s

Refer to fig. 2.7. $V_{r1} = V_1 - u_1 = V_1 - u = 81.67 - 36.75 = 44.92$ m/s $(\because u_1 = u_2 = u)$

Also, $V_{w1} = V_1 = 81.67$ m/s

From outlet velocity triangle, we have:

$V_{r2} = V_{r1} = 44.92$ m/s

Also, $V_{r2}\cos\phi = u_2 + V_{w2} = u + V_{w2}$

or, $V_{w2} = V_{r2}\cos\phi - u = 44.92\cos 15° - 36.75 = 6.64$ m/s

Work done by the jet on the runner per second

$= \rho Q(V_{w1} + V_{w2}) \times u$ ...[Eqn (2.2)]

$= 1000 \times 2.2\,(81.67 + 6.64) \times 36.75 = 7139863$ Nm/s

$\therefore$ Power given by water to the runner = 7139863 J/s

or, $W =$ **7139.8 kW (Ans.)**

(*ii*) Hydraulic efficiency of the Pelton wheel, η_h :

$$\eta_h = \frac{2(V_{w1} + V_{w2}) \times u}{V_1^2} \quad \text{...[Eqn (2.4)]}$$

$$= \frac{2\,(81.67 + 6.64) \times 36.75}{(81.67)^2} = 0.973 \quad \text{or} \quad \mathbf{97.3\ \%\ (Ans.)}$$

Example 2.3. *A Pelton wheel is to be designed for the following specifications :*

Power (brake or shaft)	*... 9560 kW*
Head	*... 350 metres*
Speed	*... 750 r.p.m.*
Overall efficiency	*... 85%*
Jet diameter	*... not to exceed 1/6 th of the wheel diameter*

Determine the following :

(i) The wheel diameter, *(ii) Diameter of the jet, and*

(iii) The number of jets required.

Take $C_v = 0{\cdot}985$, Speed ratio = 0.45.

Solution. Shaft or brake power = 9560 kW

Head, $H = 350$ m

Speed, $N = 750$ r.p.m.

Overall efficiency, $\eta_0 = 85\%$

Ratio of jet diameter to wheel, $\dfrac{d}{D} = \dfrac{1}{6}$

Co-efficient of velocity, $C_v = 0.985$

Speed ratio, $K_u = 0.45$

(*i*) The wheel diameter, D :

Velocity of jet, $V_1 = C_v\sqrt{2gH} = 0.985\sqrt{2 \times 9.81 \times 350} = 81.62$ m/s

The velocity of wheel, $u = u_1 = u_2$

$= K_u \times \sqrt{2gH} = 0.45\sqrt{2 \times 9.81 \times 350} = 37.3$ m/s

But, $u = \dfrac{\pi DN}{60}$

$\therefore$ $37.3 = \dfrac{\pi D \times 750}{60}$, or, $D = \dfrac{37.3 \times 60}{\pi \times 750} = 0.95$ m **(Ans.)**

(ii) Diameter of the jet, d :

$$\frac{d}{D} = \frac{1}{6}$$

$$\therefore \quad d = \frac{D}{6} = \frac{0.95}{6} = 0.158 \text{ m} \quad \textbf{(Ans.)}$$

(iii) The number of jets required :

$$\text{Discharge of one jet, } q = \text{Area of jet} \times \text{velocity of jet}$$

$$= \frac{\pi}{4} d^2 \times V_1 = \frac{\pi}{4} \times 0.158^2 \times 81.62 = 1.6 \text{ m}^3/\text{s}$$

Now, $$\text{overall efficiency, } \eta_0 = \frac{\text{Shaft power}}{\text{Water power}} = \frac{9560}{wQH}$$

or, $$0.85 = \frac{9560}{9.81 \times Q \times 350} \qquad (\because w = 9.81 \text{ kN/m}^3)$$

$$\therefore \quad \text{Total discharge, } Q = \frac{9560}{0.85 \times 9.81 \times 350} = 3.27 \text{ m}^3/\text{s}$$

$$\therefore \quad \text{Number of jets} = \frac{\text{Total discharge}}{\text{Discharge of one jet}} = \frac{Q}{q} = \frac{3.27}{1.6} = 2 \text{ jets} \quad \textbf{(Ans.)}$$