3.1 CLOSED LOOP FREQUENCY RESPONSE-PERFORMANCE SPECIFICATION IN FREQUENCY-FREQUENCY RESPONSE OF STANDARD SECOND ORDER SYSTEM

The response of a system can be partitioned into both the transient response and the steady state response. We can find the transient response by using Fourier integrals. The steady state response of a system for an input sinusoidal signal is known as the **frequency response**. In this chapter, we will focus only on the steady state response.

If a sinusoidal signal is applied as an input to a Linear Time-Invariant (LTI) system, then it produces the steady state output, which is also a sinusoidal signal. The input and output sinusoidal signals have the same frequency, but different amplitudes and phase angles.

Let the input signal be -

r(t)=A sin(®$_0$t)

The open loop transfer function will be -

G(s)=G(j®)

We can represent G(j®) in terms of magnitude and phase as shown below.

G(j®)= |G(j®)| zG(j®)

Substitute, ®=®$_0$ in the above equation.

G(j®$_0$)=|G(j®$_0$)|zG(j®$_0$)

The output signal is

c(t)=A| G(j® $_0$)| sin(® $_0$t+zG(j® $_0$))

- The **amplitude** of the output sinusoidal signal is obtained by multiplying the amplitude of the input sinusoidal signal and the magnitude of G(j®) at ®=®$_0$.
- The **phase** of the output sinusoidal signal is obtained by adding the phase of the input sinusoidal signal and the phase of G(j®)G(j®) at ®=®$_0$.

Where,

- **A** is the amplitude of the input sinusoidal signal.
- **®$_0$** is angular frequency of the input sinusoidal signal.

We can write angular frequency ®$_0$ as shown below.

®0 2nfo

Here, f_0 is the frequency of the input sinusoidal signal. Similarly, you can follow the same procedure for closed loop control system.

Performance Specification in Frequency and Frequency Response of Standard Second Order System:

The frequency domain specifications are **resonant peak, resonant frequency and bandwidth**, **cut-off rate, gain marginKg, phase margin(Y)**

Consider the transfer function of the second order closed loop control system as,

$$T(s) \quad \frac{C(s)}{R(s)} \quad s^2 + 2\,S\,(jd_n s + (JO\wedge$$

Substitute, s=j® in the above equation.

$$\frac{\omega_n^2}{O'\,{}^{0\,)2}\ \_\ {}^{2\,S}\ {}^{\circ}n(j^{0)}\ \_\ {}^{w}r.}$$

$$T(j\omega) = \frac{\omega_n}{-\omega^2 + 2j\delta\omega_n\omega + \omega_n^2}$$

$$T(j\omega) = \frac{1}{\left(1 - \frac{\omega^2}{\omega_n^2}\right) + j\left(\frac{2\delta\omega}{\omega_n}\right)}$$

Let, ® / $®_n$=u Substitute this value in the above equation.

$$T(J0) \qquad (\ \_\ u^2) \qquad j\,(2\ S_U)$$

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Magnitude of T(j®) is -

$$M = \backslash\ T(jo)\ | = \quad \sqrt{(1-u^2)^2 + (2\delta u)^2}$$

Phase of T(j®) is -

$$< T\,(j\ 0) = \_\ t\ an\ \blacksquare * \blacksquare(\frac{\quad}{1 - u^2})$$

Resonant Frequency:

It is the frequency at which the magnitude of the frequency response has peak value for the first time. It is denoted by $®_r$. At ®=$®_r$, the first derivate of the magnitude of T(j®) is zero.

Differentiate M with respect to u.

$$\frac{dM}{du} \quad \frac{1}{2} \quad (1 - u^2)^2 + (2 Su)^2 \, 1 \, {}_2^{-3} \, [4u(u^2 - 1 + 2 S^2)$$

Substitute, $u = u_r$ and dM / du==0 in the above equation.

$$\frac{1}{(1 - u^2)^2 + (2 Su)^2]} \, {}_2^{-3} \, [4u(u^2 - 1 + 2 S^2)]$$

$$u_r = V\,1 - 2 S^2$$

Substitute, $u_r = ®_r / ®_n$ in the above equation.

$$®_r = ®_n V\,1 - 2 S^2$$

Resonant Peak:

It is the peak (maximum) value of the magnitude of T(j®). It is denoted by M_r.

At $u = u_r$, the Magnitude of T(j®) is -

$$M = |\,T(/®)\,| = \frac{1}{V(1 - u2)^2 + (2 Su_r)^2}$$

Substitute, $u_r = V\,1 - 2 S^2$ and $1 - u_{j?} = 2 S^2$ in the above equation.

$$M_r = \frac{1}{V\,1 - S^2}$$

Resonant peak in frequency response corresponds to the peak overshoot in the time domain transient response for certain values of damping ratio 55. So, the resonant peak and peak overshoot are correlated to each other.

Bandwidth:

It is the range of frequencies over which, the magnitude of T(j®) drops to 70.7% from its zero frequency value.

At ®=0 , the value of u will be zero.

Substitute, u=0 in M.

$$M=1$$

Therefore, the magnitude of T(j®) is one at ®=0.

At 3-dB frequency, the magnitude of T(j®) will be 70.7% of magnitude of T(j®) at ®=0.

i.e., at $® = ®_B$, M=0.707(1)=^=

1 1

M =—= , V2 7(1 - uO^2 + (2 Su_b)

2 — (1 - u_b)^2 + (2 S)$^2u_b^2$

Let, $u^2_b = x$

-(4 S ^{2} - 2) ± J(4S^2 - 2)^2 + 4

x~ 2

Consider only the positive value of x.

x = 1 - 2S^2 + V(2-4 S^2 +4 S^4

Substitute, x=u $_b$=® $_b$ /® $_n$ (7 7 7)

®^{2}

J(2-4 S^2 + 4 S^4

® b — 1 — 2 S^2 + ^(2 — 4 S^2 + 4 S^4

Bandwidth $®_b$ in the frequency response is inversely proportional to the rise time t_r in the time domain transient response.

Cut-off rate:

The slope of the log-magnitude curve near the cutoff frequency is called cut-off rate

Gain Margin, K_g:

Gain margin is defined as the value of gain to be added to system in order to bring the system to the verge of instability.

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Gain Marqin, K_n = — -------- r-
(/<v)

| ( )| ( )

Phase Margin (Y):

The phase margin is obtained by adding 180^0 to the phase angle 9 of the open loop transfer function at the gain cross over frequency

Phase margin y = $180^0 + p_{gc}$