1.5 FORCE IN A SINGLY EXCITED MAGNETIC FIELD SYSTEM

Model & Analysis

The conversion of electrical energy to mechanical energy follows the law of conservation of energy. In general, the law of conservation of energy states that energy is neither created nor destroyed. Equation (1) describes the process of electromechanical energy conversion for a differential time interval dt, where dWe is the change in electrical energy, dWm is the change in mechanical energy, and dWf is the change in magnetic field energy. Energy losses in the form of heat are neglected.

$$dWe = dWm + dWf \quad (1)$$

If the electrical energy is held constant, the dWe term is zero for Equation (1). The differential mechanical energy, in the form of work, is the force multiplied by the differential distance moved. The force due to the magnetic field energy is shown in Equation (2). The negative sign implies that the force is in a direction to decrease the reluctance by making the air gap smaller

$$f_m = \frac{-dW_f}{dx} \qquad (2)$$

An expression for the energy stored in the magnetic field can be found in terms of the magnetic system parameters. This expression is then substituted into Equation (2) for Wf to get an expression for the force. This derivation is shown in Appendix A. The result is Equation (3), in terms of the current, i, the constant for the permeability of free space, m0, the cross-sectional area of the air gap, Ag, the number of turns, N, and the air gap distance, x

$$f_m = \frac{i^2 \mu_0 A_g N^2}{2x^2} \qquad (3)$$

To verify this relationship in the lab, it is convenient to have an expression for the current necessary to hold some constant force. In a design, the dimensions and force are often known. So, the user of the reluctance machine needs to know how much current to supply.

Rearranging terms in Equation (3) yields Equation (4).

$$i(x) = \sqrt{\frac{f_m \cdot 2x^2}{\mu_0 A_g N^2}} \quad (4)$$

Sample Calculations

For the simple magnetic system of Figure 3.7, the current necessary to suspend the armature can be calculated using Equation (4).

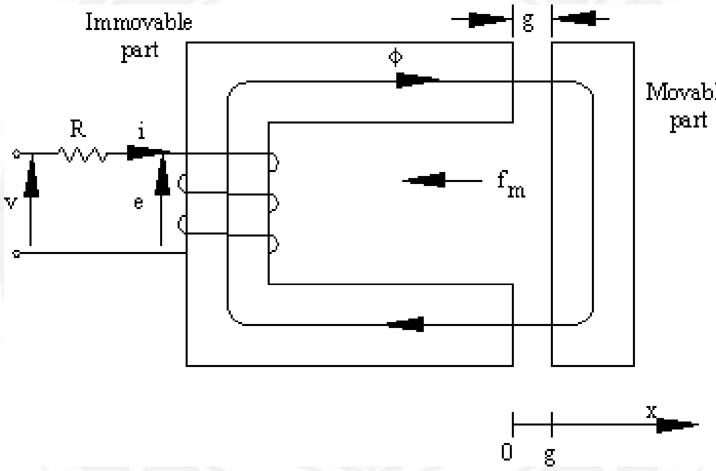

Figure 1.6.1 Electromechanical System

[Source: "'Electric Machinery Fundamentals" by Stephen J. Chapman*, Page: 231]*

For an air gap length of 0.12 mm, an air gap cross sectional area of 1092 mm2, and a 230 turn coil the current required to just suspend the 12.5 newton armature is

$$i(0.12\text{mm}) = \sqrt{\frac{(12.5\text{newton}) \cdot 2 \cdot (0.00012\text{m})^2}{\left(4 \cdot \pi \cdot 10^{-7} \frac{\text{Henry}}{\text{m}}\right) \cdot (1.092 \cdot 10^{-3} \text{m}^2) \cdot 230^2}} = 100\text{mA} \quad 5$$

Derivation of Magnetic Field Energy and Magnetic Force

Let Wf be the energy stored in a magnetic field

$$W_f = \int e \cdot i dt$$

$$e = \frac{d\lambda}{dt}$$

where l is flux linkages

$$W_f = \int \frac{d\lambda}{dt} \cdot i dt = \int i d\lambda = \int \frac{\lambda}{L} \cdot d\lambda = \frac{1}{2} \cdot \frac{\lambda^2}{L} = \frac{1}{2} \cdot i^2 \cdot L(x)$$

$$L(x) = \frac{N^2}{\Re} = \frac{N^2}{\frac{x}{\mu_0 \cdot A_g}} = \frac{\mu_0 \cdot A_g \cdot N^2}{x}$$

L(x) is the inductance as a function of the air gap length, x.

where Ag is the area of the air gap. The magnetic force is

$$f_m = -\frac{1}{2} i^2 \cdot \frac{dL(x)}{dx} = \frac{i^2 \mu_0 A_g N^2}{2x^2}$$

Force in A Multiply Excited Magnetic Field System

For continuous energy conversion devices like Alternators, synchronous motors etc., multiply excited magnetic systems are used. In practice ,doubly excited systems are very much in use.

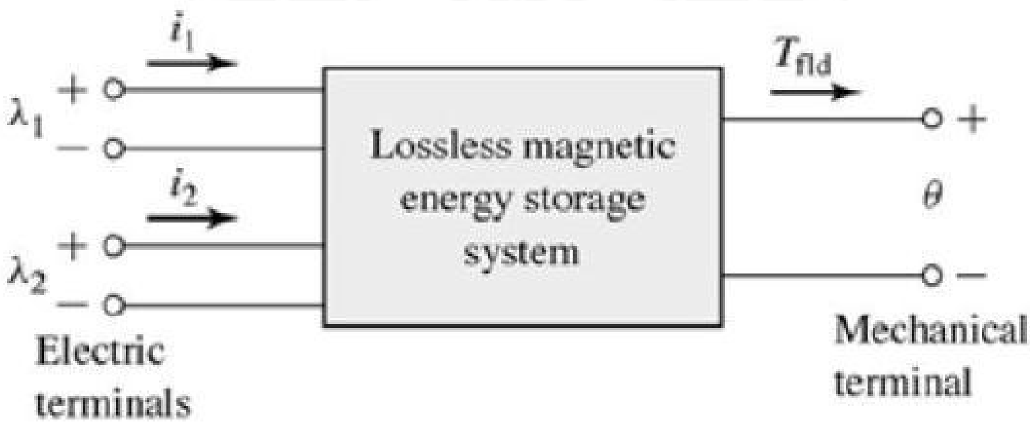

Figure 1.6.2 Electromechanical System

[Source: "Electric Machinery Fundamentals" by Stephen J. Chapman, *Page: 232]*

The Figure shows doubly excited magnetic system. This system has two independent sources of excitations. One source is connected to coil on stator while other is connected to coil on rotor.

Let i1 = Current due to source 1 i2 = Current due to source 2

= Flux linkages due to i1

= Flux linkages due to i2

= Angular displacement of rotor

Tf = Torque developed

Due to two sources, there are two sets of three independent variables

i.e. (,) or (i1,i2,)

Case:1 Independent Variables , i.e. i1,i2, From the easier analysis it is known,

Tf = ….Currents are Variables .... (1)

While the field energy is,

Wf(,) = +...............(2)

Now let L11 = Self inductance of stator L22 = Self inductance of rotor

L12= L21 = Mutual inductance between stator and rotor

= L11 i1 + L12 i2 ....(3)

And = L12 i1 + L22 i2 ....(4)

Solve equation (3) and (4) to express i1 and i2interms of and as and are independent variables. Multiply equation (3) by L12 and equation (4) by L11,

L12 =L11L12 i1+L12 i2

and L11 =L11L12 i1+L11L22 i2

Subtracting the two ,

L12 - L11 = L12 i2 - L11L22 i2

= [L12 - L11L22]i2