5.1 Transfer Function for DC Motor

Consider a separately excited DC motor with armature voltage control. In armature voltage control field current is constant but armature voltage is varied.

The figure at the right represents a DC motor attached to an inertial load The voltages applied to the field and armature sides of the motor are represented by V_f and V_a. The resistances and inductances of the field and armature sides of the motor are represented by R_f, L_f, R_a, and L_a. The torque generated by the motor is proportional to i_f and i_a the currents in the field and armature sides of the motor.

$$\boxed{T_m = K\, i_f\, i_a}$$

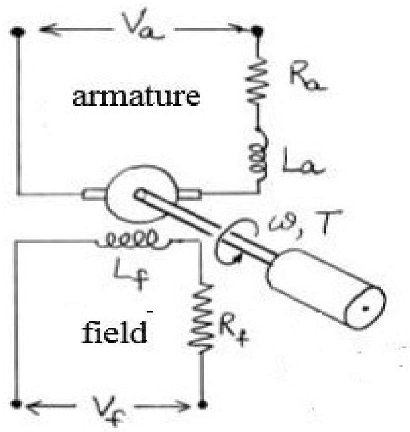

Figure 5.1.1 Speed Controller

(Source: "Fundamentals of Electrical Drives" by G.K.Dubey,page-142)

Field-Current Controlled:

In a field-current controlled motor, the armature current i_a is held constant, and the field current is controlled through the field voltage V_f. In this case, the motor torque increases linearly with the field current. We write

$$T_m = K_{mf}\, i_f$$

For the field side of the motor the voltage/current relationship is

$$\begin{aligned} V_f &= V_R + V_L \\ &= R_f i_f + L_f \left(di_f / dt \right) \end{aligned}$$

The transfer function from the input voltage to the resulting current is found by taking Laplace transforms of both sides of this equation.

$$\boxed{\frac{I_f(s)}{V_f(s)} = \frac{(1/L_f)}{s+(R_f/L_f)}} \qquad \text{(1st order system)} \qquad (1.3)$$

The transfer function from the input voltage to the resulting motor torque is found by combining equations (1.2) and (1.3).

$$\boxed{\frac{T_m(s)}{V_f(s)} = \frac{T_m(s)}{I_f(s)}\frac{I_f(s)}{V_f(s)} = \frac{(K_{mf}/L_f)}{s+(R_f/L_f)}} \qquad \text{(1st order system)} \qquad (1.4)$$

So, a step input in field voltage results in an exponential rise in the motor torque.

An equation that describes the rotational motion of the inertial load is found by summing moments

$$\sum M = T_m - cw = J\dot{w} \qquad \text{(counterclockwise positive)}$$

or

$$\boxed{J\dot{w} + cw = T_m}$$

$$\boxed{\frac{w(s)}{T_m(s)} = \frac{(1/J)}{s+(c/J)}} \qquad \text{(1st order system)} \qquad (1.5)$$

Combining equations (1.4) and (1.5) gives the transfer function from the input field voltage to the resulting speed change

$$\boxed{\frac{w(s)}{V_f(s)} = \frac{w(s)}{T_m(s)}\frac{T_m(s)}{V_f(s)} = \frac{(K_{mf}/L_f J)}{(s+c/J)(s+R_f/L_f)}} \qquad \text{(2nd order system)} \qquad (1.6)$$

Finally, since $w = dq/dt$, the transfer function from input field voltage to the resulting rotational position change is

$$\boxed{\frac{q(s)}{V_f(s)} = \frac{q(s)}{w(s)}\frac{w(s)}{V_f(s)} = \frac{(K_{mf}/L_f J)}{s(s+c/J)(s+R_f/L_f)}} \qquad \text{(3rd order system)} \qquad (1.7)$$