3.2 **BODE PLOT**

The Bode plot or the Bode diagram consists of two plots:

- Magnitude plot
- Phase plot

In both the plots, x-axis represents angular frequency (logarithmic scale). Whereas, yaxis represents the magnitude (linear scale) of open loop transfer function in the magnitude plot and the phase angle (linear scale) of the open loop transfer function in the phase plot.

The **magnitude** of the open loop transfer function in dB is -

M=20 log |G(j®)H(j®)|

The **phase angle** of the open loop transfer function in degrees is -

^=zG(j®)H(j®)

Note - The base of logarithm is 10.

Basic of Bode Plots

The following table shows the slope, magnitude and the phase angle values of the terms present in the open loop transfer function. This data is useful while drawing the Bode plots.

Type of term	**G(j®)H(j®)**	**Slope(dB/dec)**	**Magnitude (dB)**	**Phase angle(degrees)**
Constant	K	0	20 logK	0
Zero at origin	j®	20	20 log®	90
‘n’ zeros at origin	(j®)^n	20 n	20 n log®	90 n
Pole at origin	1/j®	-20	-20 log®	-90 or 270
‘n’ poles	1 / (j®)^n	-20 n	-20n log®	-90n or 270n

at origin				
Simple zero	1+j®$_r$	20	0 for ®<1/r 20 log®$_r$ for ® >1 /r	0 for ®<1/r 90 for ®>1 /r
Simple pole	1 / 1+j®r	-20	0 for ® < 1/r -20 log®$_r$ for ®>1/r	0 for ® <1/r -90 or 270 for ® >1 /r
Second order derivative term	®^{2}n(1 -®^{2}/®^2_n +2j5® / ®$_n$)	40	40 log®$_n$ for ®<®$_n$ 20 log(25®^2_n) for ®=®$_n$ 40 log® for ® >®$_n$	0 for ®<®$_n$ 90 for ®=®$_n$ 180 for ®>®$_n$
Second order integral term	1 / ®^2_n(1 -®^{2}/®^2_n+2j 5®/®$_n$)	-40	-40 log®$_n$ for ®<®$_n$ -20 log(25®^2_n) for ®=®$_n$ -40 log® for ®>®$_n$	-0 for ®<®$_n$ -90 for ®=®$_n$ -180 for ®>®$_n$

Consider the open loop transfer function G(s) H(s)=K.

Magnitude M=20 log K dB

Phase angle ^=0 degrees

If K=1, then magnitude is 0 dB.

If K>1, then magnitude will be positive.

If K<1, then magnitude will be negative.

The following figure 3.2.1 & 3.2.2 shows the corresponding Bode plot.

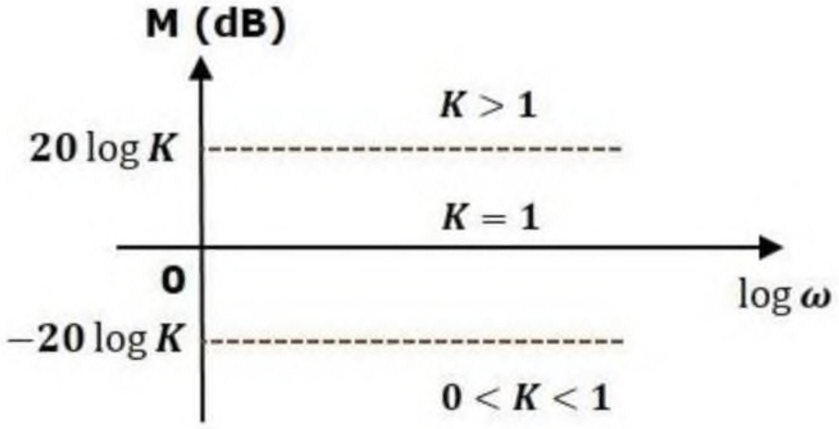

Figure 3.2.1: magnitude plot of open loop transfer function

[Source: "Control System Engineering" by Nagoor Kani, page-3.11]

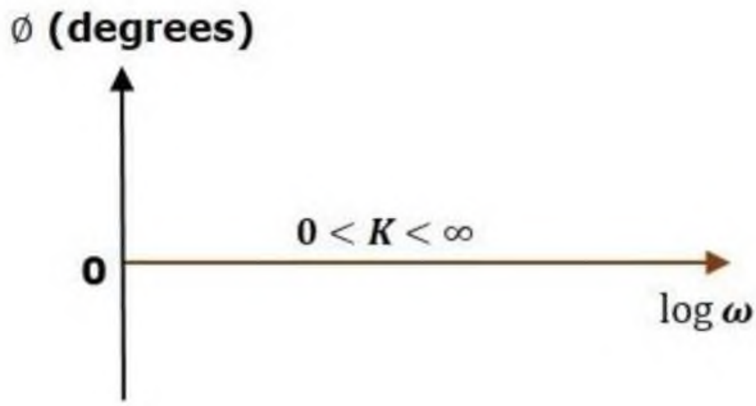

Figure 3.2.2: phase plot of open loop transfer function

[Source: "Control System Engineering" by Nagoor Kani, page-3.11]

The magnitude plot is a horizontal line, which is independent of frequency. The 0 dB line itself is the magnitude plot when the value of K is one. For the positive values of K, the horizontal line will shift 20 log K dB above the 0 dB line. For the negative values of K, the horizontal line will shift 20 log K dB below the 0 dB line. The Zero degrees line itself is the phase plot for all the positive values of K. Consider the open loop transfer function G(s)H(s)=s.

Magnitude M=20 log® dB

Phase angle ^=90^0

At ®=0.1 rad/sec, the magnitude is -20 dB.

At ®=1rad/sec, the magnitude is 0 dB.

At ®=10 rad/sec, the magnitude is 20 dB.

The following figure 3.2.3 & 3.2.4 shows the corresponding Bode plot.

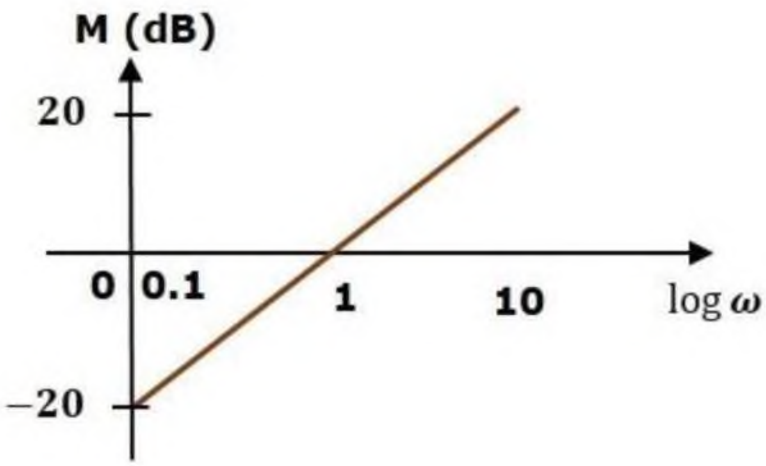

Figure 3.2.3: magnitude plot of open loop transfer function

[Source: "Control System Engineering" by Nagoor Kani, page-3.11]

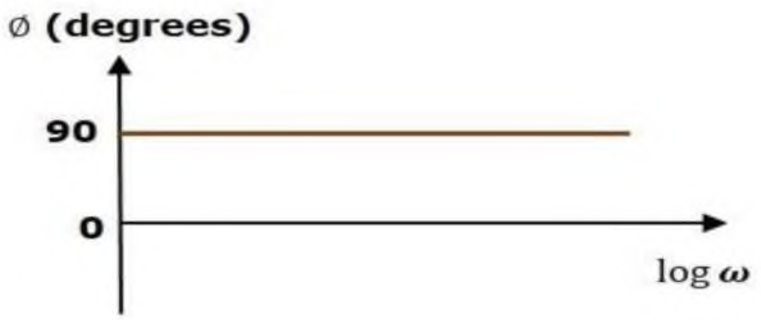

Figure 3.2.4: phase plot of open loop transfer function

[Source: "Control System Engineering" by Nagoor Kani, page-3.11]

The magnitude plot is a line, which is having a slope of 20 dB/dec. This line started at ®=0.1 rad/sec having a magnitude of -20 dB and it continues on the same slope. It is touching 0 dB line at ®=1 rad/sec. In this case, the phase plot is 90^0 line.

Consider the open loop transfer function G(S)H(S)=1+ST.

Magnitude M = 2 0 *I o g* 1 + ®^{2}r^2 dB

Phase angle ^=tan-1®T degrees

For ® < 1T , the magnitude is 0 dB and phase angle is 0 degrees.

For ® > 1T , the magnitude is 20 log®T dB and phase angle is 90^0.

The following figure shows the corresponding Bode plot.

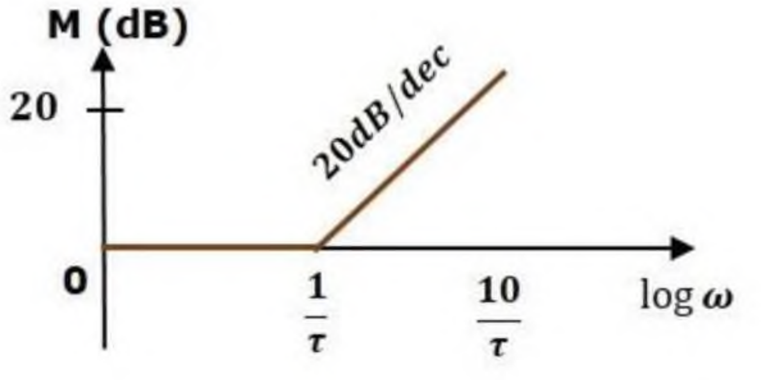

Figure 3.2.5: Magnitude plot of open loop transfer function for 1+ST

[Source: "Control System Engineering" by Nagoor Kani, page-3.11]

Qi (degrees)

90

O

10
T

log to

Figure 3.2.6: phase plot of open loop transfer function for 1+ST

[Source: "Control System Engineering" by Nagoor Kani, page-3.11]

The magnitude plot is having magnitude of 0 dB upto ®=1 /T rad/sec. From ®=1 /T rad/sec, it is having a slope of 20 dB/dec. In this case, the phase plot is having phase angle of 0 degrees up to ®=1 /T rad/sec and from here, it is having phase angle of 90^0. This Bode plot is called the **asymptotic Bode plot**.

As the magnitude and the phase plots are represented with straight lines, the Exact Bode plots resemble the asymptotic Bode plots. The only difference is that the Exact Bode plots will have simple curves instead of straight lines.

Similarly, you can draw the Bode plots for other terms of the open loop transfer function which are given in the table.

Rules for Construction of Bode Plots

Follow these rules while constructing a Bode plot.

- Represent the open loop transfer function in the standard time constant form.
- Substitute, s=j® in the above equation.

- Find the corner frequencies and arrange them in ascending order.
- Consider the starting frequency of the Bode plot as 1/10th of the minimum corner frequency or 0.1 rad/sec whichever is smaller value and draw the Bode plot upto 10 times maximum corner frequency.
- Draw the magnitude plots for each term and combine these plots properly.
- Draw the phase plots for each term and combine these plots properly.

Note - The corner frequency is the frequency at which there is a change in the slope of the magnitude plot.

Example

Consider the open loop transfer function of a closed loop control system

■' = ~~(s + Z + 5)~~

Let us convert this open loop transfer function into standard time constant form.

G(s)H(s) $\frac{}{(I+^{1})^{5}\ (h^{1})}$

G(s)W(s) $(l^{+1})\ <5^{+1})$

_ _ 10s

So, we can draw the Bode plot in semi log sheet using the rules mentioned earlier.

Stability Analysis using Bode Plots

From the Bode plots, we can say whether the control system is stable, marginally stable or unstable based on the values of these parameters.

- Gain cross over frequency and phase cross over frequency
- Gain margin and phase margin

Phase Cross over Frequency

The frequency at which the phase plot is having the phase of -180^0 is known as **phase cross over frequency**. It is denoted by $®_{pc}$. The unit of phase cross over frequency is **rad/sec**.

Gain Cross over Frequency

The frequency at which the magnitude plot is having the magnitude of zero dB is known as **gain cross over frequency**. It is denoted by $®_{gc}$. The unit of gain cross over frequency is

rad/sec.

The stability of the control system based on the relation between the phase cross over frequency and the gain cross over frequency is listed below.

- If the phase cross over frequency ®$_{pc}$ is greater than the gain cross over frequency ®$_{gc}$, then the control system is **stable**.
- If the phase cross over frequency ®$_{pc}$ is equal to the gain cross over frequency ®$_{gc}$, then the control system is **marginally stable**.
- If the phase cross over frequency ®$_{pc}$ is less than the gain cross over frequency ®$_{gc}$, then the control system is **unstable**.

Gain Margin

Gain margin GM is equal to negative of the magnitude in dB at phase cross over frequency.

G M = *2* 01 o g(j-)

Where, M_{pc} is the magnitude at phase cross over frequency. The unit of gain margin (GM) is **dB**.

Phase Margin

The formula for phase margin PM is

PM=180^0+^$_{gc}$

Where, ^$_{gc}$ is the phase angle at gain cross over frequency. The unit of phase margin is **degrees**.

The stability of the control system based on the relation between gain margin and phase margin is listed below.

- If both the gain margin GM and the phase margin PM are positive, then the control system is **stable**.
- If both the gain margin GM and the phase margin PM are equal to zero, then the control system is **marginally stable**.
- If the gain margin GM and / or the phase margin PM are/is negative, then the control system is **unstable**