Modified Euler's Method:

This method is known as predictor-corrector method. Here the slope is approximated at the middle of the interval.

The algorithm for predictor formula is

$$y_{i+1} = y_i + hf(x_i, y_i) \quad \text{...(7.62)}$$

and the algorithm for the corrector formula is

$$y_{i+1} = y_i + \frac{h}{2}[f(x_i, y_i) + f(x_{i+1}, y_{i+1})] \quad \text{...(7.63)}$$

From above it is obvious that, the predictor formula is the same as the Eulers' method where we obtain y_{i+1} at x_{i+1} with the initial value (or initial condition) of y_i at x_i. Then we correct the value of y_{i+1} thus obtained with the help of corrector formula.

Step by Step Solution of Swing Equation:-

The swing equation is

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} = \frac{1}{M}(P_i - P_m \sin\delta) \quad ....(69)$$

Its solution gives a plot of δ versus t. The swing equation indicates that δ starts decreasing after reaching maximum value, the system can be assumed to be stable. The swing equation is a non-linear equation and a formal solution is not feasible. The step by step solution is very simple and common method of solving this equation. In this method the change in δ during a small time interval Δt is calculated by assuming that the accelerating power P_a calculated at the beginning of the interval is constant from the middle of the preceding interval to the middle of the interval being considered.

Let us consider the nth time interval which begins at t = (n-1) Δt. The angular position of the rotor at this instant is δ_{n-1}(Fig. 20 c). The accelerating power $P_{a(n-1)}$ and hence, acceleration α_{n-1} as calculated at this instant is assumed to be constant from t = (n-3/2) Δt to (n-1/2) Δt.

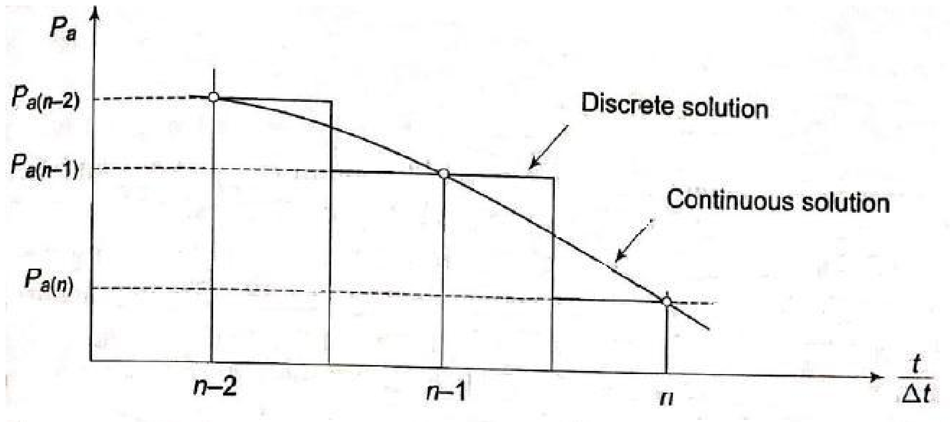
During this interval the change in rotor speed can be written as

$$\Delta\omega_{n-\frac{1}{2}} = (\Delta t)\alpha_{n-1} = \frac{\Delta t}{M} P_{a(n-1)} \quad .... (70)$$

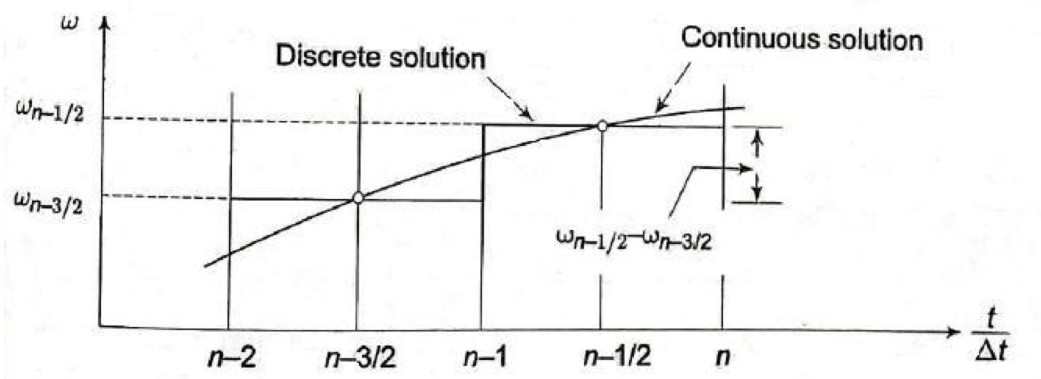
Thus, the speed at the end of nth interval is

$$\omega_{n-\frac{1}{2}} = \omega_{n-\frac{3}{2}} + \Delta\omega_{n-\frac{1}{2}} \quad .... (71)$$

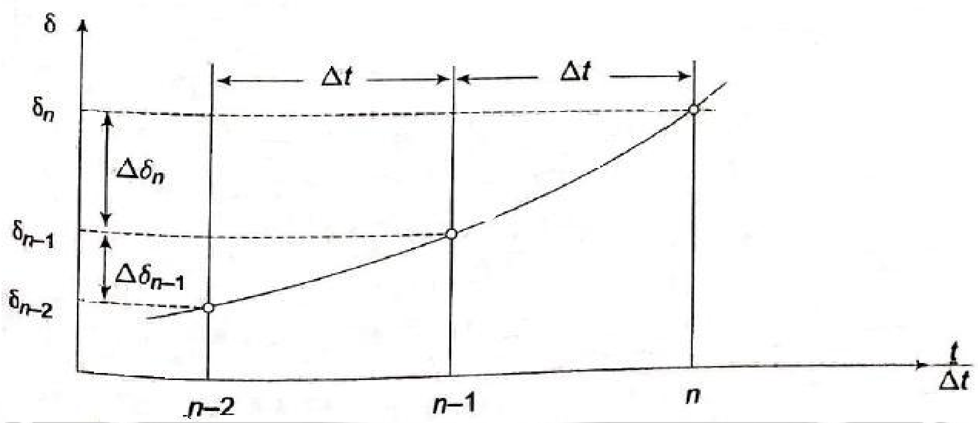
Assume the change in speed occur at the middle of one interval, i.e., t=(n-1)Δt which is same the same instant for which the acceleration was calculated. Then the speed is assumed to remain constant till the middle of the next interval as shown in Fig. 18(b). In other words, the speed assumed to be constant at the value $\omega_{n-\frac{1}{2}}$ throughout the nth interval from t = (n-1) Δt to t = n Δt.



(a)



(b)



(c)

(Fig. 20 Step by step solution of swing equation)

The change in angular position of rotor during nth time interval is

$$\Delta\delta_n = (\Delta t)\omega_{n-\frac{3}{2}} \qquad \text{.. (72)}$$

And the value of δ at the end of nth interval is

$$\delta_n = \delta_{n-1} + \Delta\delta_n \quad \text{…………………………………. (73)}$$

This is shown in Fig. 20 (c). Substituting equation (70) into equation (71) and the result in equation (72) leads to

$$\Delta\delta_n = (\Delta t)\omega_{n-\frac{3}{2}} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \quad \text{…………………….... (74)}$$

By analogy with equation (72)

$$\Delta\delta_{n-1} = (\Delta t)\omega_{n-\frac{3}{2}} \quad \text{…………………………………. (75)}$$

Substituting the value of $\omega_{n-\frac{3}{2}}$ from equation (75) into equation (74)

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \quad \text{…..…………………… (76)}$$

Equation (76) gives the increment in angle δ during any interval (say nth) in terms of the increment during (n-1) th interval.

During the calculations, a special attention has to be paid to the effects of discontinuities in the accelerating power P_a which occur when a fault is applied or cleared or when a switching operation takes place. If a discontinuity occurs at the beginning of an interval then the average of the values of P_a before and after the discontinuity must be used. Thus, for calculating the increment *in* δ occuring in the first interval after a fault is applied at t=0, equation (76) becomes

$$\Delta\delta_1 = \frac{(\Delta t)^2}{M} \cdot \frac{P_{a0+}}{2} \quad \text{……………………………………. (77)}$$

Where P_{a0+}, is the accelerating power immediately after occurrence of the fault. Immediately before the occurrence of fault, the system is in steady state with $P_{a0-} = 0$ and the previous increment in rotor angle is zero.

<u>Multimachine stability Studies</u>:-

The equal-area criterion cannot be used directly in systems where three or more machines are represented, because the complexity of the numerical computations increases with the number of machines considered in a transient stability studies. To ease the system complexity of system modeling, and thereby computational burden, the following assumptions are commonly made in transient stability studies:

1. The mechanical power input to each machine remains constant.
2. Damping power is negligible.
3. Each machine may be represented by a constant transient reactance in series with a constant transient internal voltage.

4. The mechanical rotor angle of each machine coincides with δ.
5. All loads may be considered as shunt impedances to ground with values determined by conditions prevailing immediately prior to the transient conditions.

   The system stability model based on these assumptions is called the **classical stability model**, and studies which use this model are called **classical stability studies**.

Consequently, in the multi-machine case two preliminary steps are required.

1. The steady-state prefault conditions for the system are calculated using a production-type power flow program.
2. The prefault network representation is determined and then modified to account for the fault and for the postfault conditions.

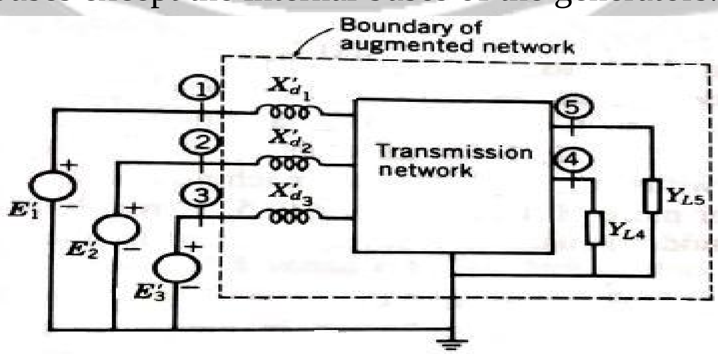
The transient internal voltage of each generator is then calculated using the equation

$$E = V_t + jX'_d I \quad \text{.. (80)}$$

Where V_t is the corresponding terminal voltage and I is the output current. Each load is converted into a constant admittance to ground at its bus using the equation

$$Y_L = \frac{P_L - jQ_L}{|V|^2} \quad \text{.. (81)}$$

Where $P_L - jQ_L$ the load and |VL| is is the magnitude of the corresponding bus voltage. The bus admittance matrix which is used for the prefault power-flow calculation is now augmented to include the transient reactance of each generator and the shunt admittance of each load, as shown in Fig. 21. Note that the injected current is zero at all buses except the internal buses of the generators.



(Fig. 21 Augmented network of a power system)

In the second preliminary step the bus admittance matrix is modified to correspond to the faulted and post fault conditions. During and after the fault the power flow into the network from each generator is calculated by the corresponding power angle equation. For example, in Fig. 21 the power output of generator 1 is given by

$$P_{e1} = |E'_1|^2 G_{11} + |E'_1||E'_2||Y_{12}|\cos(\delta_{12} - \theta_{12}) + |E'_1||E'_3||Y_{13}|\cos(\delta_{13} - \theta_{13}) \quad (82)$$

Where δ_{12} equals $\delta_1 - \delta_2$. Similar equations are written for P_{e2} and P_{e3} using the Y_{ij} elements of the 3X3 bus admittance matrices appropriate to the fault or postfault condition. The P_{ei} expressions form part of the equations

$$\frac{2H_i}{\omega_s}\frac{d^2\delta_i}{dt^2} = P_{ii} - P_{ei} \qquad i=1, 2, 3 \quad (83)$$

Which represent the motion of each rotor during the fault and post fault periods. The solutions depend on the location and duration of the fault, and Y_{bus} resulting when the faulted line is removed.

Factors Affecting Transient Stability:-

Various methods which improve power system transient stability are

1. Improved steady-state stability
    a) Higher system voltage levels
    b) Additional transmission line
    c) Smaller transmission line series reactance
    d) Smaller transfer leakage reactance
    e) Series capacitive transmission line compensation
    f) Static var compensators and flexible ac transmission systems (FACTs)
2. High speed fault clearing
3. High speed reclosuer of circuit breaker
4. Single pole switching
5. Large machine inertia, lower transient reactance
6. Fast responding, high gain exciter
7. Fast valving
8. Breaking resistor